Bargaining Order in a Multi-Person Bargaining Game*

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Abstract

This paper studies a complete-information bargaining game with one buyer and multiple sellers of different "sizes" or bargaining strengths. The bargaining order is determined by the buyer. If the bargaining has a finite horizon, there is a unique subgame perfect equilibrium outcome in which the buyer purchases in order of *increasing* size – from the smallest to the largest. With an infinite horizon, there is a unique equilibrium outcome with the same bargaining order if the sellers are of sufficiently different sizes. With an infinite horizon, there are multiple equilibria with different bargaining orders if the sellers have similar sizes. However, if the buyer can commit to the order in which she bargains with the sellers, she will commit to the order of increasing size.

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1 Introduction

Consider a scenario in which a real estate developer must acquire land from multiple sellers. The sellers' lots are of different sizes with a larger lot giving a higher flow of payoffs to its owner. Such situations are quite common. For example, in Chongqing, China, the construction of a particular retail mall required 280 properties of different residents. The project was suspended for three years because one out of the 280 owners refused to sell his property to the developer.[1] Columbia University's expansion plan in West Manhattanville is another prominent example. The 17-acre project was worth 6.3 billion dollars, and the land is acquired from 67 separate property owners. The entire negotiation lasted over a long period from years 2002 to 2010, and the negotiation on the last three properties alone took

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[1] The negotiation began in 2004, and eventually the owner sold his property in 2007. See French (2007) or "Nail House in Chongqing Demolished", *China Daily*, April 3, 2007.

more than three years.[2] What should the buyer (developer) do when she needs to purchase land from multiple sellers who own lots of different sizes? In particular, which seller should she bargain with first, the one with a large lot or a small lot? This paper examines the corresponding non-cooperative bargaining game. We find that the buyer should bargain with the seller of the smallest lot first, especially when the sizes of the lots are quite different. This paper does not try to explain the delay in the examples above. Delays occurred even when there was only one seller remaining in the first example, perhaps due to incomplete information.[3]

While the model studied here is couched in the language of a single developer negotiating with multiple sellers, it is applicable to a variety of other bargaining scenarios. For example, consider an airline that must bargain with two separate unions, pilots and flight attendants, in order to end a strike. Both unions are necessary for the airline to operate, but their outside options differ.[4] Which union should the firm negotiate with first? A similar question can be asked about the negotiation between a manufacturer and a group of upstream suppliers producing parts at different costs.[5] Our model also applies to the case in which a good, in order to reach the buyer, needs to pass a sequence of intermediaries with different transaction costs.[6] The key characteristics common to these scenarios are: the one-to-many aspect of the negotiation; the fact that an agreement with *all* sellers is necessary to reap any economic gains; and finally, the "size" differences among the sellers.

In this paper, bargaining strength is measured by the size of the inside/outside options available to a seller when bargaining with the buyer.[7] A seller with a large lot is stronger than a seller with smaller lot in the sense that, in equilibrium, the price received by the large seller is higher than that received by the small seller. There are other notions of bargaining strength, of course. For instance, one may measure bargaining strength by how patient a seller is and different sellers may have different discount rates. Alternatively, it may be related to the likelihood of making initial offers (see, for instance, Li 2010).

It is useful to begin with a simple example. Consider a scenario with one developer and two farmers. All parties share a discount factor of $\delta \in (0, 1)$. Farmer 1 owns a large lot of land that produces $(1 - \delta)v_1$ units of harvest in each period; farmer 2 owns a small lot of land that produces $(1 - \delta)v_2$ units of harvest in each period. We assume v_1 is greater than v_2. The land does not produce any harvest once it is sold to the developer. The developer must purchase both lots to build a mall that produces $1 - \delta$ units of profit in each period. It is easy to see that the present value of all harvests is v_1 for farmer 1 and v_2 for farmer 2.

[2] See Williams (2008), Chung (2009) and Sieff (2010).

[3] See, for instance, Admati and Perry (1987) for how incomplete information can lead to delay.

[4] An outside option is the payoff that a player receives if he leaves the negotiation.

[5] Bargaining between a manufacturer and its upstream suppliers is discussed, for example, in Blanchard and Kremer (1997), and Bedrey (2009).

[6] Manea (2013) discusses this example along with others in a study of a different topic on intermediation in networks.

[7] An inside option is the payoff received by a seller while negotiations are ongoing (see, for example, Muthoo 1999). The analysis in Sections 2 and 3 focus on inside options, but our qualitative results would not be affected if sellers had outside options instead. Section 4.1 discusses outside option in more details.

The present value of the total profit of the mall is 1.

Negotiations are sequential. In any period, the developer negotiates with only one farmer. The developer first offers a price, which the farmer may accept or reject. If the offer is accepted, the developer proceeds to negotiate with the other farmer in the next period (in a standard two-player alternating offer bargaining game). If the offer is rejected, the farmer makes a counter-offer in the next period, which the developer may accept or reject. If the developer accepts this offer, she proceeds to negotiate with the other farmer. If the developer rejects the offer, she picks a farmer, who could be the same one as in the previous period, and negotiates with him in the same fashion, and so on. Which farmer should the developer bargain with first?

For the purpose of this example, assume that the bargaining ends after a finite number of periods, and we consider the extreme case in which the number of periods goes to infinity. Our result can be best illustrated in the extreme case with δ close to 1. A larger discount factor could stand for shorter periods. As δ becomes larger, the harvest within each period becomes smaller, while the present value of harvests remains the same.

Since the developer can pick any remaining farmer to negotiate with, there is no restriction on the choice of bargaining order. However, there is a unique subgame perfect equilibrium outcome, in which the developer purchases from farmer 2 first and then from farmer 1. The equilibrium prices are explained below. The payment to the first farmer is a sunk cost to the buyer. Therefore, after farmer 2 sells his land, the surplus is $1 - v_1$, which is the difference between the value of the mall and the value of farmer 1's harvests. If δ converges to 1, farmer 1 receives $\delta/(1+\delta) = 1/2$ of the surplus as in the Rubinstein bargaining game.[8] This implies that a surplus of $\frac{1}{2}(1 - v_1)$ is paid to farmer 1, so he sells at a price of $v_1 + \frac{1}{2}(1 - v_1)$.[9] Excluding the price for farmer 1, the remaining value of mall is $\frac{1}{2}(1 - v_1)$. As a result, the total surplus for farmer 2 and the buyer is $\frac{1}{2}(1 - v_1) - v_2$, which is also the difference between the remaining value of the mall and the value of farmer 2's harvests. Similarly, farmer 2 and the buyer split this surplus equally as in the Rubinstein bargaining game. Therefore, a surplus of $\frac{1}{2}\left[\frac{1}{2}(1 - v_1) - v_2\right]$ is paid to farmer 2, and the buyer's payoff is $\frac{1}{2}\left[\frac{1}{2}(1 - v_1) - v_2\right] = \frac{1}{4} - \frac{v_2}{4} - \frac{v_1}{2}$. In contrast, if the buyer bargains with seller 1 first instead, she would receive a payoff of $\frac{1}{4} - \frac{v_1}{4} - \frac{v_2}{2}$, which is lower in comparison, since $v_1 > v_2$.

Our model builds on the model of Cai (2000) by introducing endogenous bargaining order and asymmetric sellers. His model is the extreme case of our infinite-horizon game if the farmers do not receive harvest. The bargaining order is fixed and rotates among the sellers in his paper. He finds multiple stationary subgame perfect equilibrium outcomes, and that delay can occur in some of them. In contrast, the sellers are asymmetric and the bargaining order is endogenous in our game, resulting in a *unique* subgame perfect equilibrium outcome.

Several papers have the feature that the bargaining orders are endogenously determined, but they are determined in a restricted way. Perry and Reny (1993) allow each player to decide when to make an offer, which implicitly allows for different bargaining orders. Stole

[8]See Rubinstein (1982).

[9]The equilibrium prices are calculated according to Lemma 1.

and Zwiebel (1996), Noe and Wang (2004) and Bedrey (2009) study bargaining orders in finite-horizon bargaining games. Chatterjee and Kim (2005) focus on the bargaining orders, in which the buyer does not switch to another seller before an agreement. The literature on agenda formation also discusses orders, but the orders have a different meaning: sequences of different issues or tasks.[10] Contrary to our paper, this literature suggests that the most important issue should be discussed first.[11]

Li (2010) also allows endogenous bargaining orders, but his paper is very different from ours. In his paper, a seller's bargaining strength is measured by his likelihood to make the first offer in each bargaining. He finds multiple equilibria and that any selling order can be sustained. In our paper, a seller's bargaining strength is measured by the size of his inside option, and the bargaining game has a unique equilibrium outcome. Krasteva and Yildirim (2012) also use the likelihood of making an offer to represent bargaining strength. They study bargaining order in a two-period game, and find that, in the presence of uncertain payoffs for the sellers, the buyer may prefer different bargaining orders depending on the sellers' likelihood to make the first offer.

The seller holdout problem also has complementary sellers, as in our setup.[12] In this literature, it is argued that incomplete information often leads to holdout. However, holdout is not important in our setup because our bargaining is of complete information and the bargaining order is endogenously determined.[13]

The rest of the paper is organized as follows. Section 2 studies the bargaining game with a finite horizon, while Section 3 studies the bargaining game with an infinite horizon. Section 4 discusses alternative assumptions and applications. Section 5 concludes the paper.

2 Bargaining with a Finite Horizon

Our model is a non-cooperative and complete-information bargaining game with endogenous bargaining orders and asymmetric sellers. The game has $N+1$ players including one buyer, B, and a set of sellers, $\{1, 2, \cdots, N\}$. Each seller (he) has one lot of land, and the buyer (she) must purchase every lot in order to build a mall. In other words, the lots are perfect complements for the buyer.

All the players share the same discount factor $\delta \in (0, 1)$.[14] Each seller receives a constant flow of harvests. Seller i's harvest for one period is of value $v_i(1-\delta)$, and is received at the end of each period while the land is still in his possession. In the literature of bargaining, the harvest is referred as to seller i's inside option. If seller i does not sell his land, he would receive harvests for infinitely many periods, so the present value of the harvests is $v_i(1-\delta)\sum_{t=1}^{+\infty}\delta^{t-1} = v_i$. We assume that $v_1 > v_2 > \cdots > v_N > 0$. If every unit area of land

[10] See, for example, Fershtman (1990), Winter (1997), Reinhard and Matthias (2001) and Flamini (2007).

[11] See, for example, Winter (1997) and Flamini (2007).

[12] See, for example, Mailath and Postlewaite (1990), Menezes and Pitchford (2004) and Chowdhury and Sengupta (2012).

[13] Holdout may arise if the bargaining order is exogenously given. See Cai (2000) for example.

[14] If the sellers are different only in their discount factors, it can be verified that the buyer would be indifferent among different orders because they all give her the same payoff.

is equally productive, the assumption implies that seller i's land is of larger size than seller $(i+1)$'s. The mall produces a constant profit in each period, and the present value in period 1 of all profits of the mall is normalized to 1.

2.1 Timing

We first consider a bargaining game with a finite horizon of $T \geq N$ periods, and then in Section 3 consider the case with an infinite horizon. The bargaining game is a natural extension of the Rubinstein bargaining game.[15] More precisely, at the beginning of the game, the buyer picks one seller and bargains with him for one round. Each round can have one or two periods. In the first period, the buyer suggests a price to the seller, the seller then decides to either accept it or reject it. If the seller accepts, this round ends with only one period. Otherwise, the seller suggests another price in the second period, which the buyer must either accept or reject. If an agreement is reached, the buyer pays the seller the agreed price right away and the seller leaves the game permanently before the harvests of the period are realized. If the seller's suggestion is rejected, the buyer picks one of the remaining sellers, who may be the same or different to the previous one, and bargains with him in the next round in the same fashion. Note that the length of each round is endogenous and depends on the strategies. At the end of each period, every remaining seller receives a harvest from his land.

Note that there is no restriction on the bargaining order in the sense that the buyer can choose any remaining seller to bargain with. Let $G(i, j, t)$ denote the t-period two-seller game in which the player i suggests a price to player j in the first period. Let $G(i, t)$ denote the t-period one-seller game in which the buyer makes an offer to seller i in the first period. Figure 1 illustrates the two-seller bargaining game for $T = 3$, where $p_{i,t}^n$ denotes the buyer's offer to seller i in the first period of the n-seller game with t periods, and $q_{i,t}^n$ denotes seller i's offer to the buyer in the first period of the n-seller game with t periods.

Two features of the model are important. First, the buyer bargains with only one seller at a time. This bargaining feature is popular in reality, especially when it is costly to communicate with all the sellers at the same time. In the example by Coase (1960), a railway company has to bargain with the farmers along a railway track. It is difficult to make simultaneous offers to multiple farmers when they are located far away from each other. Because of its relevance in reality, this feature is also being examined in several studies on bargaining such as Cai (2000, 2003) and Noe and Wang (2004). Second, the payments are made immediately after the corresponding agreements. In other words, the contracts between the buyer and sellers are cash-offer contracts. These contracts are widely used by real estate developers.[16] In addition, pharmaceutical or electronics companies often need to purchase multiple patent

[15]However, our results are not specific to the setup with alternating offers. For instance, suppose that in each period, the buyer or the seller is selected with some probability to make an offer, which the other accepts or rejects. We can obtain the same equilibrium bargaining order. In addition, if the buyer and the seller chosen by her undertake Nash bargaining instead of making alternating offers, we can get the same results on bargaining order as well.

[16]See "Nail House in Chongqing Demolished", *China Daily,* April 3, 2007.

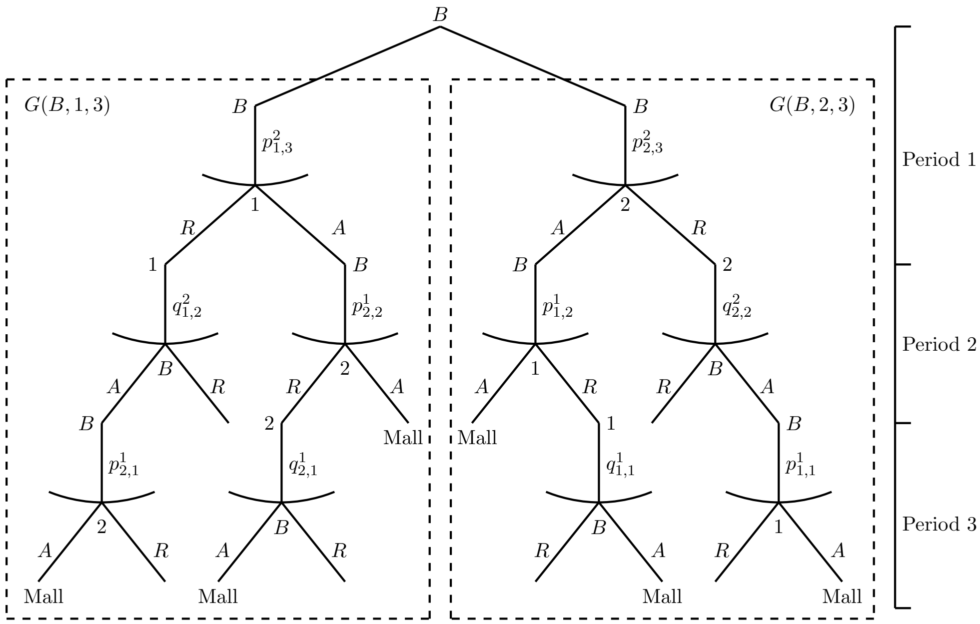

Figure 1: Bargaining in Three Periods

licenses controlled by different owners in order to launch a product. Delays in payment may be unacceptable as negotiation could reveal some key ideas behind the patent. This is especially true for software patents. Moreover, it would be difficult to settle with a union on strike by a contract contingent on future events.[17] Because their prevalence, cash-offer contracts are also being also studied in recent bargaining literature.[18] The consequences of relaxing these assumptions are discussed in Section 4.

2.2 Payoffs

An outcome is denoted as $(p_1, p_2, ..., p_N, t_1, t_2, ..., t_N)$, where seller i sells his land at price p_i in period t_i. If seller i never sells his land, t_i is infinity and p_i is zero. The present value of t periods of seller i's harvests is denoted as

$$H_{i,t} \equiv v_i(1-\delta)[1+\delta+...+\delta^{t-1}].$$

Note that a seller continues to receive harvests after period T if the mall is not built.

Given an outcome, seller i's payoff is

$$\pi_i = H_{i,t_i-1} + \delta^{t_i-1} p_i, \tag{1}$$

[17]In a strike bargaining started in 2005, the Northwest Airlines settled with the Aircraft Mechanics Fraternal Association (AMFA), which represented aircraft mechanics, janitors, and aircraft cleaners, in November 2006, then settled with the flight attendant union half a year later in May 2007. The agreements with AMFA were not conditional on the later negotiation. See Pongrace (2007) for more details.

[18]See Krishna and Serrano (1996) and Cai (2000, 2003).

where the first term is the present value of the harvests prior to land sale, and the second term is the present value of the buyer's payment. Since there is no harvest when the buyer owns the land, the buyer's payoff is

$$\pi_B = \delta^{\max(t_1,\cdots,t_N)-1} - \sum_{i=1}^{N} \delta^{t_i-1} p_i, \tag{2}$$

where the first term is the present value of the mall and the second term is the present value of the payments to the sellers. We assume that the buyer initiates the bargaining only if she is going to receive a positive payoff.[19] Seller i's surplus is $\pi_i - H_{i,t_i-1}$, which is his payoff minus the present value of his harvests. The buyer's surplus is her payoff because she does not have harvest.

The assumption that the buyer cannot reap the harvests from the land represents the fact that the buyer cannot fully utilize the land (as the sellers could) before the mall is built. Take the land purchasing case in Chongqing as an example. The sellers received utility by living in their houses, but the buyer could not receive the same amount of total utility even if she owns the houses.

2.3 Equilibrium

Throughout the paper, "equilibrium" refers to the subgame perfect equilibrium. We first present a general result of the N-seller game in Proposition 1, then discuss it in the context of two sellers. In particular, Proposition 2 establishes a unique equilibrium outcome in a two-seller game, and Lemma 1 discusses the equilibrium as the horizon goes to infinity.

Proposition 1 *Suppose the horizon T is long enough and*

$$\sum_{i=1}^{N} \left(\left(\frac{\delta}{1+\delta} \right)^{N-i} v_i \right) < \left(\frac{\delta}{1+\delta} \right)^{N-1}, \tag{3}$$

then the N-seller game has a unique equilibrium outcome, and the buyer always purchases from the smallest remaining seller first. Moreover, as T goes to infinity, every player's equilibrium surplus converges to a positive value under (3) and to zero if $\sum_{i=1}^{N} \left(\left(\frac{\delta}{1+\delta} \right)^{N-i} v_i \right) > \left(\frac{\delta}{1+\delta} \right)^{N-1}$.

The proof is in Appendix A. Proposition 1 establishes a unique equilibrium outcome given (3) and a long horizon. Without (3) or the long horizon, we cannot exclude the possibility of multiple equilibrium outcomes. However, Proposition 1 implies that, even in the presence of multiple equilibrium outcomes, the difference in each player's surplus across the outcomes vanishes as the horizon goes to infinity. If there are only two sellers, Proposition 2 shows a

[19]This assumption eliminates the situations in which the buyer is indifferent to the bargaining orders as they all lead to a zero payoff for her.

stronger result than Proposition 1. It establishes a unique equilibrium outcome without a long horizon and (3).

Proposition 2 *The two-seller game with a finite horizon $T \geq 2$ has a unique equilibrium outcome. Moreover, the buyer always purchases from the smaller seller first.*

Proof. We sketch a proof below, with the full proof provided in Appendix A. In particular, Claim 1 characterizes the equilibrium prices in a one-seller game, and it shows that the buyer's payoff with an even horizon is higher than that with an odd horizon. This is because the buyer offers in the last period if T is even. For the same reason, the equilibria in a two-seller game for an even horizon are quite different to those with an odd horizon, so we discuss them separately.

Suppose the horizon T is even, then the first agreement may not be in the first period. However, we can show by induction that if the mall is ever built, the first purchase must be from the smaller seller. In particular, Claim 2 shows that this is true for $T = 2$. Assuming the smaller seller sells first if $T = 2t$, we show in Claims 3 to 8 that purchasing from seller 2 dominates purchasing from seller 1 if $T = 2t + 2$. That is, the buyer receives a higher payoff if she bargains with the smaller seller first. Hence the smaller seller sells first for any even $T \geq 2$.

Suppose the horizon T is odd. It is sufficient to show Claim 9, which states that if the mall is built, the smaller seller sells in the first period. We prove this claim by induction. First, Claim 10 shows that it is true for $T = 3$. Assume Claim 9 is true for $T = 2t - 1$, we prove the claim for $T = 2t + 1$ in two cases. In the first case, suppose the mall is built if the horizon is $T = 2t - 1$. Then, Claims 11 to 15 show that the mall is also built if the horizon is $T = 2t + 1$, and seller 2 must sell in the first period. In the second case, suppose the mall is not built if the horizon is $T = 2t - 1$. Then, Claim 16 proves Claim 9 for the horizon of $T = 2t + 1$. ■

It is worth mentioning that there may be multiple equilibria associated with the same unique outcome. We can see this in the following example.

Example 1 *Consider a two-seller game with $T = 4$, $v_1 = 0.3$, $v_2 = 0.1$ and $\delta = 0.8$.*

By backward induction, we can verify that there are at least two equilibria. In one equilibrium, the buyer bargains with seller 2 first. No agreement is reached in the first two periods. Then, seller 2 sells first in period 3 at a price of v_2, and seller 1 sells in period 4 at a price of v_1. To see why there is a delay of two periods, suppose that seller 2 sells in period 2. His price should not be lower than v_2. After seller 2 sells, seller 1 sells in period 3 at a price of $v_1 + \delta(1 - v_1) = 0.86$.[20] Therefore, the buyer's payoff evaluated in period 2 is no more than $\delta(1 - 0.86) - v_2 = 0.01$. However, the buyer could wait one period and get a payoff of $1 - v_1 - v_2 = 0.6$, so her payoff in period 2 should be at least $0.6\delta = 0.48$, which is

[20]The calculation is based on Claims 1 and 2.

higher than 0.01. This is a contradiction. Similarly, we can show that no seller would sell in the first or second period.

In another equilibrium, the buyer bargains with seller 1 first. No agreement is reached in the first two periods. Then, sellers 2 and 1 sell in period 3 and 4 as in the first equilibrium. Each seller sells at the same time for the same price in both equilibria, so the outcome is identical. However, since no agreement can be reached in the first two periods, the buyer may bargain with either seller first in equilibrium.

More generally, if the horizon is $2t > 4$ in the above example, there is no agreement until only two periods remain. That is, there are $2t-2$ periods of delay. Therefore, if the horizon $T = 2t$ goes to infinity, the equilibrium payoff converges to zero for the buyer, v_1 for seller 1 and v_2 for seller 2. As a result, every player's surplus converges to zero because of the long delay. If there is a small fixed cost of bargaining, the mall would not be built in this example. Hence the more interesting equilibria are those in which the players earn positive surplus if T goes to infinity. Lemma 1 characterizes such equilibria.

Lemma 1 *Consider the two-seller game and suppose*

$$\frac{1}{1+\delta}\delta(1-v_1) - v_2 < 0, \tag{4}$$

then every player's limit equilibrium surplus as the horizon T goes to infinity is positive. Moreover, seller 2 sells in the first period and

$$\lim_{T\to\infty} p_{2,T}^2 = v_2 + \frac{\delta}{1+\delta}\left[\frac{1}{1+\delta}\delta(1-v_1) - v_2\right]; \tag{5}$$

seller 1 sells in the second period and

$$\lim_{T\to\infty} p_{1,T-1}^1 = v_1 + \frac{\delta}{1+\delta}(1-v_1); \tag{6}$$

and the buyer's payoff satisfies

$$\lim_{T\to\infty} \pi_{B,T} = \frac{1}{1+\delta}\left[\frac{1}{1+\delta}\delta(1-v_1) - v_2\right]. \tag{7}$$

Suppose $\frac{1}{1+\delta}\delta(1-v_1) - v_2 > 0$, then every player's surplus in equilibrium converges to zero as T goes to infinity.

The proof of the lemma is in Appendix A. Let us explain the intuition for the lemma. After the first purchase, if the mall is built, its value is 1. In contrast, if the mall is not built, the present value of all seller 1's harvests is v_1. Therefore, the surplus is the difference between these values, $1 - v_1$. The buyer and seller 1 split this surplus as if they are in the Rubinstein bargaining game. More precisely, the surplus paid to seller 1 is $\frac{\delta}{1+\delta}(1-v_1)$ according to (6). Therefore, with the surplus for seller 1 excluded, the mall would be worth $\frac{1}{1+\delta}(1-v_1)$ in period 2, or $\delta\frac{1}{1+\delta}(1-v_1)$ in period 1. As a result, the agreement with seller 2 produces a surplus of $\delta\frac{1}{1+\delta}(1-v_1)-v_2$, which is the remaining value of the mall minus the value of seller 2's harvests. It is easy to see from (5) and (7) that the buyer and seller 2 also split the surplus $\frac{\delta}{1+\delta}(1-v_1)-v_2$ as in the Rubinstein bargaining game, where (4) guarantees that this surplus is positive.

Let us explain why the buyer prefers to purchase from the smaller seller first. Suppose the buyer purchases from the larger seller first. Then, by the argument above, the buyer would receive a payoff of $\frac{1}{1+\delta}\left[\frac{1}{1+\delta}\delta(1-v_2)-v_1\right]$, which is lower than that if she purchases from the smaller seller first.

Suppose $\frac{1}{1+\delta}\delta(1-v_1)-v_2<0$ is satisfied, as T converges to infinity, the equilibrium payoffs converge to zero for the buyer, v_1 for seller 1 and v_2 for seller 2, and every player earns zero surplus in the limit. Since we do not restrict the value of δ, all our results below hold if δ is almost 1.

In our model explicated above, the heterogeneity in the sellers' inside options allows the selling orders to affect the surplus after the first purchase. However, this may not be true for other types of heterogeneity, and there could be multiple equilibrium outcomes. For example, Li (2010) considers the heterogeneity in sellers' probabilities to make the first offer, so no matter who sells first, the surplus after the first purchase is the same. He finds multiple equilibrium outcomes with different bargaining orders.

3 Bargaining with an Infinite Horizon

We consider the same bargaining game as above except this time with an infinite horizon. Hereafter, the N-seller game with an infinite horizon is referred to as the "N-seller game". Let $\Gamma(i)$ represent the one-seller game between seller i and the buyer who makes the first offer. We use $\Gamma(j,j')$ to denote the two-seller game in which player j suggests a price to player j' in the first period. Figure 2 illustrates the game tree for the two-seller game.

After the first agreement in an N-seller game, the remaining subgame is an $(N-1)$-seller game. Because of this recursive structure, we focus on which sellers the buyer bargains with before the first purchase. More precisely, given an equilibrium, a bargaining order in an N-seller game is an infinite sequence of sellers. In the equilibrium, the buyer bargains with the tth seller in the sequence if no agreement has been reached after the buyer bargains with the first $t-1$ sellers in the sequence. After the first purchase, the buyer follows a bargaining order in the resulting $(N-1)$-seller game, and so on.

In the rest of this section, we first discuss the infinite-horizon game in two cases. In the first case, where the sellers are of sufficiently different sizes, we have a unique equilibrium outcome. In the second case, where the sellers are of similar sizes, we have multiple equilibrium outcomes and a few examples are provided. Then we discuss the case in which the buyer has the option to commit to a bargaining order, and find that the buyer will commit to the order of increasing size.

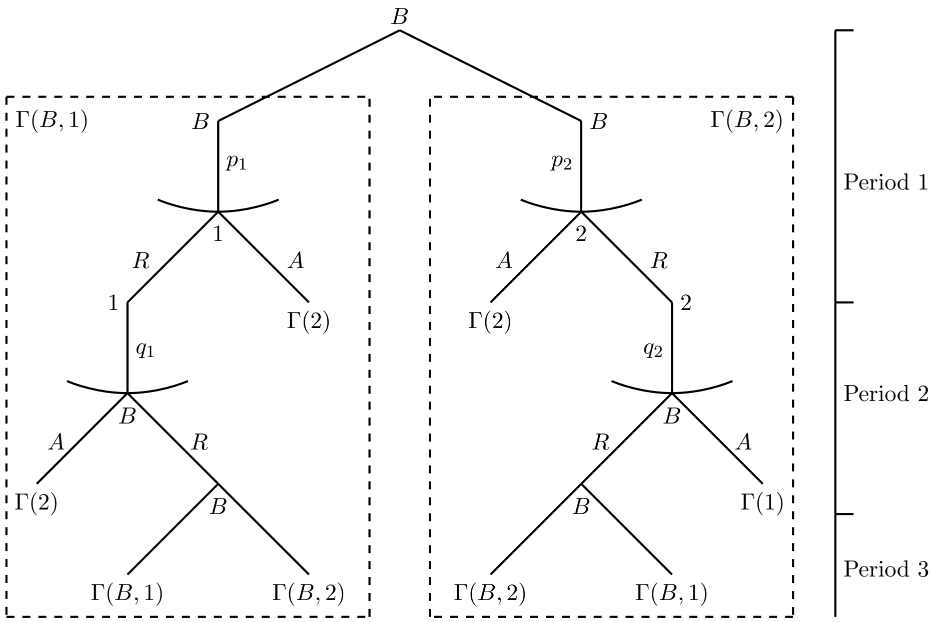

Figure 2: Bargaining with an Infinite Horizon

3.1 Sufficiently Different Sizes

We first present the result of the N-seller game in Proposition 3, and discuss it in the two-seller game.

Proposition 3 *In an N-seller game with an infinite horizon, if the mall is profitable as in*

$$\sum_{i=1}^{N}\left(\left(\frac{\delta}{1+\delta}\right)^{N-i} v_i\right) < \left(\frac{\delta}{1+\delta}\right)^{N-1}, \tag{8}$$

and the sellers' sizes are sufficiently different as in

$$\begin{aligned} &\left(1-\frac{\delta}{\delta+1}\right)(v_{n-1}-v_n)+\sum_{i=1}^{n-1}\left((v_i-v_{i+1})\sum_{j=n-i}^{n-1}\left(\frac{\delta}{1+\delta}\right)^{j}\right) \\ > \quad &\left(\frac{\delta}{\delta+1}\right)^{n-1} - v_n\sum_{i=1}^{n}\left(\frac{\delta}{1+\delta}\right)^{i-1}, \end{aligned} \tag{9}$$

for $n = 2, ..., N$, then there is a unique equilibrium outcome and a unique bargaining order where the buyer bargains with the smallest remaining seller until an agreement is reached.

The proof of the proposition is in Appendix B. Condition (9) ensures that, when the remaining sellers are $1, 2, ..., i$, seller i sells first. If it is violated for some i, different bargaining orders may arise in equilibria. We discuss the different bargaining orders in Section 3.2. In the reminder of this section, we discuss Proposition 3 in the two-seller game. In particular, we first discuss the subgame in which the buyer bargains with seller 2 first; then study the subgame in which the buyer bargains with seller 1 first. Finally we compare the two scenarios and prove Proposition 4, which is a special case of Proposition 3 in the two-seller game.

Since the payment to the first seller is a sunk cost to the buyer, the subgame after the first purchase is a one-seller game between the buyer and the remaining seller. Therefore, let us first examine the one-seller game with a land value of v_i.

In the one-seller game, the buyer has to bargain with the same seller in every period, so the game is simply a two-person alternating bargaining game with inside options only available to the seller. Then, the unique equilibrium of such a game is characterized in Lemma 2, which is a straightforward adaptation of Proposition 6.1 in Muthoo (1999).

Lemma 2 *Define p_i^1 and q_i^1 as*

$$\begin{aligned} p_i^1 &= v_i + \frac{\delta}{1+\delta}(1-v_i), \\ q_i^1 &= 1 - \frac{\delta}{1+\delta}(1-v_i). \end{aligned} \tag{10}$$

In the one-seller game between the buyer and seller i, there is a unique equilibrium. In the equilibrium,

i) the seller offers a price of q_i^1 and accepts a price no less than p_i^1,

ii) the buyer offers a price of p_i^1 and accepts a price no more than q_i^1.

The superscript denotes the number of sellers in the game where the variables are considered, but it is omitted if there is no ambiguity. Similar to Proposition 6.1 in Muthoo (1999), the buyer and the seller should be indifferent between accepting and rejecting the other player's offer in the unique equilibrium. Note that p_i^1, q_i^1 solve equation system

$$p_i = H_{i,1} + \delta q_i. \tag{11}$$

$$1 - q_i = \delta(1-p_i). \tag{12}$$

where equation (11) means that the buyer makes an offer such that the seller is indifferent between accepting and rejecting, and equation (12) means that the seller asks for a price such that the buyer is indifferent between accepting and rejecting.

Equation (10) implies that the equilibrium price p_i^1 is always higher than the seller's value of the land v_i. Moreover, the equilibrium payoffs are $v_i + \frac{\delta}{1+\delta}(1-v_i)$ for the seller and $\frac{1}{1+\delta}(1-v_i)$ for the buyer, so they split the surplus of $1-v_i$, as in the Rubinstein bargaining game.

Lemma 2 characterizes the unique equilibrium after the first purchase in the two-seller game, so we only focus on the strategies *before* the first purchase in the rest of this section. In particular, Lemma 3 characterizes all the equilibria, and Proposition 4 shows that the corresponding outcome is unique. Since the payment to the first seller is a sunk cost to the buyer, the subgame after the first purchase is a one-seller game between the buyer and the remaining seller. As we already know the unique equilibrium in the one-seller game, the following lemma focuses on the strategies *before* the first purchase.

Lemma 3 *Define p_2^2, q_2^2 as*

$$p_2^2 = v_2 + \frac{\delta}{1+\delta}[\delta(1-p_1^1) - v_2], \tag{13}$$

$$q_2^2 = \delta\left(1-p_1^1\right) - \frac{\delta}{1+\delta}\left[\delta(1-p_1^1) - v_2\right], \tag{14}$$

and p_1^2, q_{B1}^2 as

$$p_1^2 = H_{1,3} + \delta^3 p_1^1, \tag{15}$$

$$q_{B1}^2 = \delta(1-p_2^1) - \delta(\delta(1-p_1^1) - p_2^2).$$

If

$$\delta v_1 + (1+\delta)\, v_2 < \delta \tag{16}$$

and

$$v_1 - v_2 > \frac{\delta}{1+\delta} - \frac{1+2\delta}{1+\delta} v_2 \tag{17}$$

for any (q_1^2, p_{B1}^2) such that $q_1^2 > q_{B1}^2$ and $p_{B1}^2 < p_1^2$, the strategies below constitute an equilibrium in the game $\Gamma(B,2)$:

i) seller 2 suggests a price of q_2^2 and accepts a price no less than p_2^2,

ii) seller 1 suggests a price of q_1^2 and accepts a price no less than p_1^2,

iii) the buyer bargains with seller 2 until an agreement is reached; suggests a price p_2^2 to seller 2 and p_{B1}^2 to seller 1; accepts a price no more than q_2^2 from seller 2 and no more than q_1^2 from seller 1.

Proof. We can verify that $\left(p_2^2, q_2^2\right)$ satisfies

$$p_2^2 = H_{2,1} + \delta q_2^2, \tag{18}$$

$$\delta\left(1-p_1^1\right) - q_2^2 = \delta(\delta(1-p_1^1) - p_2^2). \tag{19}$$

Substituting p_2^2 and q_2^2 into (1) and (2) gives the equilibrium payoffs for the buyer and seller 2:

$$\pi_B^* = \frac{1}{1+\delta}\left[\delta\frac{1}{1+\delta}(1-v_1) - v_2\right], \tag{20}$$

$$\pi_2^* = v_2 + \frac{\delta}{1+\delta}\left[\delta\frac{1}{1+\delta}(1-v_1) - v_2\right]. \tag{21}$$

Since Lemma 2 shows that seller 1 sells at price p_1^1 in period 2, his equilibrium payoff is

$$\pi_1^* = v_1 + \delta\frac{\delta}{1+\delta}(1-v_1). \tag{22}$$

We can verify that q^2_{B1} satisfies

$$\delta(1 - p^1_2) - q^2_{B1} = \delta(\delta(1 - p^1_1) - p^2_2). \tag{23}$$

The left hand side of (23) is the buyer's payoff if she accepts price q^2_{B1} from seller 1, and the right hand side is $\delta\pi^*_B$, so the buyer is indifferent between accepting q^2_{B1} and rejecting it. As a result, the buyer accepts prices no higher than q^2_{B1} from seller 1. However, (17) implies $q^2_{B1} < v_1$, so seller 1 cannot afford any price that the buyer would accept, therefore seller 1 suggests q^2_1 which is below the buyer's threshold of acceptance, q^2_{B1}, and does not deviate.

The left hand side of (15) is seller 1's payoff if he accepts p^2_1. If seller 1 rejects p^2_1, he receives π^*_1 with two periods of delay, which is the right hand side of (15). Hence, the equation means that seller 1 is indifferent between accepting p^2_1 and rejecting it. As a result, seller 1 accepts a price no less than p^2_1. However, (17) implies that accepting any price above p^2_1 is not profitable for the buyer. Therefore the buyer offers p^2_{B1}, which is below seller 1's threshold of acceptance, and does not deviate.

As in (19), the buyer is indifferent between accepting q^2_2 from seller 2 and rejecting it. Hence the buyer would not change her threshold of acceptance, q^2_2, and seller 2 would not change his offer q^2_2.

As in (18), seller 2 is indifferent between accepting p^2_2 from the buyer and rejecting it. Hence the seller would not deviate from her threshold of acceptance, p^2_2, and the buyer would not deviate from his offer p^2_2. ■

Note that the equilibrium prices are p^1_1 for seller 1 and p^2_2 for seller 2, which are the same as those in Lemma 1. This means that, when the horizon goes to infinity, the equilibrium outcome of a finite horizon game converges to the outcome of the finite horizon game. As a result, the intuition for Lemma 1 also applies to the game with an infinite horizon.

Proposition 4 *In a two-seller infinite-horizon game, if the mall is profitable as in (16) and the sellers' sizes are sufficiently different as in (17), there is a unique equilibrium outcome and a unique bargaining order where the buyer bargains with the smaller seller until an agreement is reached.*

The proof of this proposition is in Appendix B. It is an extension of the uniqueness proof for the Rubinstein bargaining game.[21] In our model, seller 2 always sells first because the buyer receives a negative payoff if seller 1 sells first. Given the price accepted by seller 2, everything is known according to the unique equilibrium of the Rubinstein bargaining game. As a result, the price for seller 2 is the only parameter to be determined for the set of possible outcomes and therefore for the set of possible payoffs as well. Once the set of possible payoffs is characterized by the single parameter, the rest of the proof is parallel to the proof of Rubinstein bargaining game.

It is easy to see that Proposition 4 holds when δ is almost 1. Condition (17) requires that, for a fixed size of the smaller lot, the difference between the sizes is sufficiently large. Similar

[21] See Shaked and Sutton (1984).

to the discussion after Lemma 1, the buyer and seller 1 split a surplus of $\delta \frac{1}{1+\delta}(1-v_2)-v_1$ if seller 1 sells his land first. However, (17) implies that the surplus is negative, so seller 1 does not sell first as this would lead to a negative payoff for the buyer. Therefore, both the offer and counter-offer would be rejected when the buyer bargains with seller 1 first, and $\Gamma(B,1)$ would have a delay of two periods in equilibrium. Condition (17) guarantees the unique equilibrium outcome. If this condition is violated, there are multiple equilibrium outcomes with different bargaining orders, as will be shown in Section 3.2.

Let us explain the implications of Lemma 3 and Proposition 4. Lemma 3 characterizes all the equilibria, which share the same outcome $\left(p_1^1, p_2^2, 2, 1\right)$ and the same bargaining order $2, 2, \ldots$. Since the equilibrium outcome is unique according to Proposition 4, all the strategies are uniquely determined by backward induction except for q_1^2 and p_{B1}^2. As a result, Lemma 3 also describes all the equilibria of $\Gamma(B,2)$.

The game $\Gamma(B,1)$ is a proper subgame of $\Gamma(B,2)$, so the equilibria and the equilibrium outcome are inherited from Lemma 3. In particular, if there is no agreement in period 1 or 2, then the buyer chooses seller 2 to bargain with and seller 2 sells in period 3 and seller 1 sells in period 4. As a result, there is a delay of two periods and the payoffs are

$$\begin{aligned}
\pi_1^{*\prime} &= H_{1,2} + \delta^2 \pi_1^*, \\
\pi_2^{*\prime} &= H_{2,2} + \delta^2 \pi_2^*, \\
\pi_B^{*\prime} &= \delta^2 \pi_B^*.
\end{aligned}$$

This means that delays can happen if a "wrong" order is chosen. However, it can be avoided if the buyer chooses the bargaining order in which the buyer bargains with the smallest remaining seller until an agreement is reached.

Even though the bargaining game is of complete information, the "wrong" bargaining order may cause delay in agreement. Several papers have described different reasons for delay in complete information bargaining. The reason discussed above is similar to Cai (2000). On the other hand, the reasons in Haller and Holden (1990) and Fernandez and Glazer (1991) are different from those identified here. Their models are standard two-person alternating bargaining games between a labor union and a firm, but the labor union can choose between production and strike when an offer is rejected. In an equilibrium with delay, the firm would rather wait several periods to avoid the "bad" equilibrium in which the union goes on strike once disagreement occurs. In our paper, production (building the mall) is not allowed while the bargaining is ongoing. Harvests are different to production in that the sellers receive them with certainty during the bargaining process.

3.2 Similar Sizes

When the sellers are of similar sizes, (17) could be violated. The outcome in Lemma 3 is still an equilibrium outcome when (17) is violated, but there are other equilibria with different outcomes and bargaining orders. Two other equilibria are described in the following two lemmas.

First, if we exchange sellers 1 and 2 in Lemma 3, it gives us another set of equilibria.

Lemma 4 *Let* $\left(p_1^{2\prime}, q_1^{2\prime}, p_2^{2\prime}, q_{B2}^{2\prime}\right)$ *be the solution to*

$$\begin{aligned} p_1 &= H_{1,1} + \delta q_1, \\ \delta\left(1 - p_2^1\right) - q_1 &= \delta\left(\delta\left(1 - p_2^1\right) - p_1\right), \\ p_2 &= H_{2,3} + \delta^3 p_2^1, \\ \delta\left(1 - p_1^1\right) - q_{B2} &= \delta\left(\delta\left(1 - p_2^1\right) - p_1\right). \end{aligned}$$

If condition (16) is satisfied but (17) is violated, then for any $\left(q_2^{2\prime}, p_{B2}^{2\prime}\right)$ *such that* $q_2^2 > q_{B2}^{2\prime}$ *and* $p_{B2}^2 < p_2^{2\prime}$*, the strategies below constitute an equilibrium in the two-seller game without commitment:*

i) seller 1 accepts a price no less than $p_1^{2\prime}$ *and suggests a price of* $q_1^{2\prime}$*,*

ii) seller 2 accepts a price no less than $p_2^{2\prime}$ *and suggests a price of* $q_2^{2\prime}$*,*

iii) the buyer bargains with seller 1 before the first purchase; accepts a price no more than $q_1^{2\prime}$ *from seller 1 and no more than* $q_{B2}^{2\prime}$ *from seller 2; and suggests* $p_1^{2\prime}$ *to seller 1 and* $p_{B2}^{2\prime}$ *to seller 2.*

The proof is the same as the proof for Lemma 3. In the equilibrium, seller 1 sells in period 1 and seller 2 sells in period 2, and the bargaining order is $1, 1, ...$

Second, there is another set of equilibria in which no agreement is reached in the first two periods.

Lemma 5 *For the two-seller game with an infinite horizon, if (16) is satisfied, (17) is violated, and*

$$\delta\left[\delta\frac{1}{1+\delta}(1-v_1) - v_2\right] > \delta\frac{1}{1+\delta}(1-v_2) - v_1, \tag{24}$$

the following strategies constitute an equilibrium: the buyer chooses to bargain with seller 1 in the first period; everyone follows the strategies in Lemma 3 in $\Gamma(B,1)$*; everyone follows the strategies in Lemma 4 in* $\Gamma(B,2)$*.*

Proof. Lemma 3 implies that the strategies in $\Gamma(B,2)$ are an equilibrium, and Lemma 4 implies that the strategies in $\Gamma(B,1)$ are also an equilibrium. By backward induction, the buyer chooses seller 1 to bargain with first because the buyer's payoff in $\Gamma(B,1)$ is higher than in $\Gamma(B,2)$ according to (24). ■

As a result, there is no agreement in the first two periods and sellers 2 and 1 sell in periods 3 and 4 respectively, and the corresponding bargaining order is $1, 2, 2, ...$

Using the equilibrium payoffs in Lemmas 3, 4 and 5 as punishments, we can construct many other equilibria; moreover, there is a continuum of equilibria without delays.[22] However,

[22]A similar analysis is used in the three-person alternating bargaining game. See, for instance, Herrero (1984) and Osborne and Rubinstein (1990). The equilibria with delay also demonstrate different bargaining orders as in Lemma 5 and Lemma 6, though they are not presented for the consideration of space.

it is difficult to find the full characterization of equilibria or equilibrium payoffs even for the two-seller game. The difficulty is briefly explained below.

In order to find the full characterization, we need to find the minimum and maximum for each player's payoffs as in the three-person alternating bargaining game.[23] Both the selling prices and the length of delay could affect the bounds. For example, seller 2's minimum payoff could be reached through a low selling price with a shorter delay or by a higher selling price but with a longer delay. Moreover, the two factors interact with each other making the problem even more challenging. Since the maximum length of delay is very likely to be increasing in δ, the difficulty remains even when δ approaches 1.

If condition (9) is violated, an N-seller game could have multiple equilibria with different outcomes and different bargaining orders as in a two-seller game. Other equilibria can be constructed similarly as in Lemmas 4 and 5. Compared to the two-seller game, it is even more difficult to find a full characterization of the equilibria in the N-seller game. The larger number of sellers greatly increases the number of possible bargaining orders, and there is a large number of possible selling orders given each bargaining order. Moreover, when (9) is violated for some, but not all, n smaller than N, the full characterization becomes even more challenging.

3.3 Commitment to Bargaining Order

In this section, we consider a setup in which the buyer has the option to commit to a bargaining order. By comparing the cases with and without commitment, we will demonstrate that the buyer can maximize her payoff by committing to the bargaining order of increasing size.

In contrast to the case without commitment, the buyer first announces a sequence of pairs $(i_1, n_1), (i_2, n_2), ...$, where i_j is a seller and n_j is a positive integer or ∞. This sequence means that the buyer will bargain with seller i_1 for n_1 rounds (if $n_1 < \infty$) or until that seller sells his land (if $n_1 = \infty$), then she will switch to seller i_2 for n_2 rounds (if $i_1 \neq i_2$) or until seller i_2 sells his land.

Here we only focus on the bargaining orders before the first agreement, and it is represented by an infinite sequence of sellers, $i_1, i_2, ...$ where $i_t \in \{1, 2, \cdots, N\}$ for all t. Starting with the first seller in the sequence, the buyer bargains with one seller over the price in each round of bargaining. It should be emphasized that each round can have one or two periods, and the bargaining in each round is the same as in the finite-horizon game. If a seller's suggestion is rejected in one round, the bargaining moves on to the next round in which the buyer bargains with the second seller in the sequence, in the same fashion as earlier. At the end of each period, every remaining seller receives a harvest from his land.

Note that the bargaining order cannot be revised afterwards. Hereafter, the above game is referred as to the "N-seller game with commitment", or the "N-seller game" if there is no ambiguity. The bargaining order in the N-seller game specifies only the order of sellers

[23] See, for instance, Herrero (1984), Osborne and Rubinstein (1990).

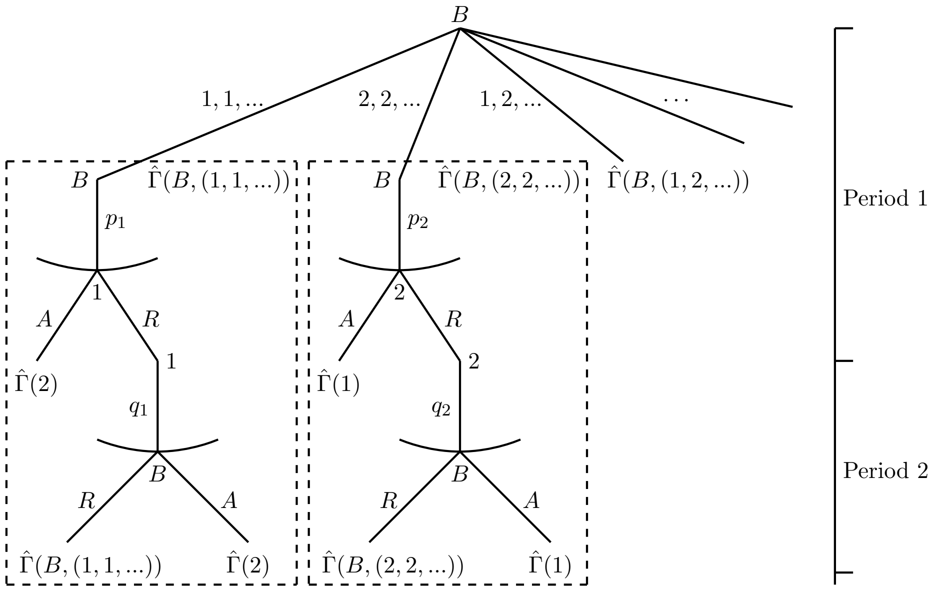

Figure 3: Bargaining with Commitment

before the first agreement. After the first agreement, the game has only $N-1$ sellers and the buyer chooses an order for the resulting $(N-1)$-seller game, and so on.

Let $\hat{\Gamma}(j,(i_1,i_2,...))$ denote the two-seller game with a *fixed* bargaining order i_1, i_2, ..., where player $j \in \{i_1, B\}$ makes the first offer. The bargaining order i_1, i_2, ... means that the buyer bargains with seller i_t in period $2t+1$ if no agreement has been achieved. For example, a bargaining order 1, 1, ... means the buyer bargains with seller 1 until he agrees. Let $\hat{\Gamma}(i)$ represent the one-seller game between seller i and the buyer who makes the first offer. Figure 3 demonstrates the game tree for the two-seller game.

3.3.1 Equilibrium

In the remainder of this section, we first present the result of the N-seller game in Proposition 5, then discuss its proof and equilibrium strategies in the two-seller game.

Proposition 5 *In the N-seller game with commitment, if the mall is profitable as in (8), there is a unique equilibrium outcome and a unique equilibrium bargaining order where the buyer bargains with the smallest remaining seller until an agreement is reached. Moreover, the buyer's equilibrium payoff with commitment is equal to the highest equilibrium payoff without commitment.*

Proof. We sketch below a proof for the case of two sellers, and the full proof is in Appendix C. The proof consists of five claims. First, in an equilibrium with agreement in the first period, any equilibrium payoff for the buyer can be reached. Second, consider the equilibria of the subgames with either bargaining order $1,1,...$ (the buyer bargains with seller 1 until he agrees) or $2,2,...$ (the buyer bargains with seller 2 until he agrees). In these equilibria, the supremum of the buyer's equilibrium payoffs, $\bar{\pi}_B$, can be approached. Third, if the buyer commits to order $2, 2, ...$, the corresponding subgame has a unique equilibrium with an agreement in the first period; when the buyer commits to order $1, 1, ...$, the resulting subgame has either a unique equilibrium with agreement in the first period or equilibria with no agreement at all. Fourth, if the buyer chooses order $1, 1, ...$, her payoff is less than $\bar{\pi}_B$. Fifth, it is only if the buyer chooses order $2, 2, ...$, that the buyer's equilibrium payoff reaches $\bar{\pi}_B$. ■

Note that, not only are the equilibrium bargaining orders the same for both finite or infinite horizons, the equilibrium prices also converge to the ones in the infinite-horizon game if the horizon goes to infinity.

It is surprising that the equilibrium outcome is unique because multiple equilibrium outcomes are prevalent in similar bargaining games.[24] For example, if the buyer follows a bargaining order that alternates between the sellers, there are multiple equilibrium outcomes by the same analysis in Theorem 1 of Cai (2000). In our model, uniqueness results because the bargaining order is endogenous. Since the buyer does not choose the alternating order in the equilibria, multiple equilibrium outcomes do not arise.

With p_2^2, q_2^2 defined in (13) and (14), the following lemma characterizes the equilibrium strategies in the two-seller game.

Lemma 6 *Under (16), the strategies below constitute an equilibrium in the game $\Gamma(B, (2, 2, ...))$: i) the buyer suggests a price of p_2^2 to seller 2 and accepts a price no more than q_2^2 from seller 2, ii) seller 2 suggests a price of q_2^2 and accepts a price no less than p_2^2.*

Proof. According to equation (18), seller 2 is indifferent between accepting and rejecting p_2^2. In particular, if seller 2 accepts p_2^2 in the current period, his payoff is the left hand side of (18). If seller 2 rejects p_2^2, he receives a harvest at the end of the current period, and accepts q_2^2 in the next period, which gives him a payoff equal to the right hand side of (18).

Similarly, according to (19), the buyer is indifferent between accepting and rejecting q_2^2. In particular, if the buyer accepts q_2^2, she pays q_2^2 to seller 2 in the current period, pays p_1^1 to seller 1 and receives the value of the mall in the next period, which gives her a payoff equal to the left hand side of (19). If the buyer rejects q_2^2, she pays p_2^2 to seller 2 in the next period, pays p_1^1 to seller 1 and receives the value of the mall two periods later, which gives her payoff on the right hand side of (19).

As a result, neither seller 2 nor the buyer would deviate in the subgame $\hat{\Gamma}(B, (2, 2, ...))$, so the lemma is proved. ■

[24] However, several papers also demonstrate a unique subgame perfect equilibrium in multi-person bargaining games. See Jun (1987), Chae and Yang (1988) and Krishna and Serrano (1996). There are multiple equilibria in our bargaining game, but all of them have the same outcome. As in the proof of Claim 19, there is no agreement in $\Gamma(B, (1, 1, ...))$ if $\delta v_2 + (1 + \delta) v_1 > \delta$. Therefore, given that any offer is rejected, it is seller 1's equilibrium strategy to offer any price no less than v_1. As a result, the subgame $\Gamma(B, (1, 1, ...))$ has many equilibria, and so does the whole game.

Notice that the outcome is the same as in Lemma 3, so the equilibrium payoffs are π_B^*, π_2^* and π_1^*, as given in (20) to (22). Note that the prices are the same as those in Lemma 1. This means that, when the horizon goes to infinity, the equilibrium outcome of a finite horizon game converges to the outcome in the finite horizon game. Therefore, the intuition for Lemma 1 also applies to the game with an infinite horizon.

4 Discussion and Applications

4.1 Inside vs. Outside Options

It is natural to use inside options (ongoing profits from farming) to represent the sizes of sellers. However, outside options can also be used to represent the different sizes of sellers. In particular, suppose a seller can also sell his land in an outside market besides accepting and rejecting when he receives an offer. If any seller chooses the outside option, the bargaining ends. In this setting, our qualitative results would not be affected. Let us demonstrate this in an example of bargaining with commitment.

Suppose that there are two sellers with outside options of 0 and $1/2$ and their discount factor is close to 1. First, suppose that the buyer bargains with the smaller seller (with outside option 0) until he agrees. After the smaller seller agrees, the buyer bargains with the larger seller (with outside option $1/2$). The surplus of this bargaining is the difference between the mall's value and the outside option of the larger seller, $1-1/2$. Since the players are patient, they split this surplus evenly, and the big seller receives a surplus of $(1-1/2)/2$. Therefore, the buyer and the smaller seller split the remaining surplus $(1-1/2)/2$ evenly, giving the buyer a surplus of $(1-1/2)/4$.

Second, suppose that the buyer bargains with the larger seller until he agrees. Similarly, the surplus after the larger seller agrees is $1-0$, and the smaller seller receives half of it, $1/2$. Since the total surplus is $1-1/2-0$, there is no surplus left for the buyer and the larger seller. As a result, the buyer prefers to bargain with the smaller seller until he agrees. The above analysis only illustrates the comparison of two bargaining orders in the bargaining game with commitment, but we can similarly derive the counterparts of other results with inside options.

4.2 Coordination Among Sellers

It is an interesting question whether the sellers gain by merging into a single agent who bargains on behalf of all sellers. The answer is they will not. Consider the example in Section 4.1. Suppose that the two players merge into one agent. Then, suppose this agent has an outside option of $1/2$, and the bargaining is a standard two-person bargaining game with an outside option. As a result, the agent receives $(1-1/2)/2$, which is half of the total surplus. However, the sellers receive a total surplus of $3/8$ if they do not merge. The main reason is that the buyer needs agreement from each seller, therefore each seller has "veto" power. If the sellers merge and there are less players with "veto" power, then their bargaining power is also reduced. Moreover, it may be beneficial for a seller to split his land and let some agents represent the different pieces.

4.3 Heterogeneous Discounting

It is a natural extension to consider players with different discount factors. For the purpose of illustration, we only consider the two-seller game with commitment. Suppose the players have discount factors, δ_1, δ_2, and δ_B for sellers 1, 2 and the buyer. Equation (10) becomes

$$p_i^1 = v_i + r_i \left(1 - v_i\right),$$

where $r_i = \left(1 - \delta_B\right) \delta_i / (1 - \delta_i \delta_B)$ and equations (18) and (19) become

$$\begin{aligned} p_2 &= v_2 \left(1 - \delta_2\right) + \delta_2 q_2, \\ \delta_B \left(1 - p_1^1\right) - q_2 &= \delta_B \left(\delta_B \left(1 - p_1^1\right) - p_2\right). \end{aligned}$$

Then, similar analysis implies that, if the buyer bargains with the smaller seller until he agrees, the equilibrium prices are $p_1^1 = v_1 + r_1 \left(1 - v_1\right)$ and $p_2^2 = v_2 + r_2 \left(\delta_B \left(1 - p_1\right) - v_2\right)$, where the buyer's payoff is $\pi_B = \left(1 - r_2\right) \left(r_1 \left(1 - v_1\right) - v_2\right)$. Similarly, if the buyer bargains with the larger seller until he agrees, the buyer's payoff is $\pi'_B = \left(1 - r_1\right) \left(r_2 \left(1 - v_2\right) - v_1\right)$. If $\delta_2 \leq \delta_1$, the buyer's payoff is higher if she bargains with seller 2 (the smaller seller) first, and our qualitative results would not be affected. However, if seller 2 is much more patient than seller 1, it may be efficient for seller 1 to sell first. This effect of allowing different discount factors could dominate the effect from "sizes", and the buyer may bargain with seller 1 until he agrees in an equilibrium.

4.4 Order of Offers

This paper assumes that the buyer makes the first offer in each round of bargaining, however, our result is not hinged upon this assumption. For instance, consider the example in Section 4.1, and suppose the sellers make the first offer in each round of bargaining. It is easy to see that the equilibrium prices and payoff are the same. Important to our results is the presence of inside/outside options. Without inside/outside options, there could be multiple equilibria with different bargaining orders (see, for instance, Cai 2003 and Li 2010).

4.5 Beyond Perfect Complementarity

Our results apply to some scenarios in which the lots are not perfect complements. For instance, consider the same setup except that there are four sellers $1, 2, 3, 4$ with values $0 < v_4 < v_3 < v_2 < v_1$. In order to build the mall, the buyer needs the land from seller 1, 2, 3 or seller 1, 2, 4. Note that the sellers are no longer perfect complements. Assume that there is a finite horizon of 3 periods. By backward induction, we can verify that the prices must be equal to the valuations because the sellers do not have time to make counter-offers.

Then, the buyer's payoff is

$$\pi_B = \delta(\delta(\delta(1 - v_i) - v_j) - v_k),$$

where k is the first seller, j is the second seller and i is the last seller. If the mall is built, it must be that $\{i, j, k\} = \{1, 2, 3\}$ or $\{1, 2, 4\}$. Comparing the two possibilities, we can show that the buyer's payoff is maximized at

$$\pi_B = \delta(\delta(\delta(1 - v_1) - v_2) - v_3)$$

Hence, if the mall is built, the buyer would choose to bargain with sellers in the order of increasing size as well. It would be a very interesting question to consider the equilibrium bargaining order with more sellers and longer horizons, but the setup would be quite different from the current version and it is better to explore it in another paper. If the lots are perfect substitutes, the buyer may prefer to bargain with the larger seller first. See, for instance, Krasteva and Yildirim (2012).

4.6 Cash-Offer vs. Contingent Contracts

This paper considers cash-offer contracts, which are prevalent in the real estate business. However, there are other bargaining situations where contingent contracts could be used. Under a contingent contract, the payments are not made until all the sellers have agreed. In contrast to our paper, if the buyer uses contingent contracts in our model, the surplus remains the same after the agreements. There are still equilibria with the bargaining order of increasing size. However, the order of increasing size may not be the only equilibrium bargaining order in this setup.[25]

4.7 Simultaneous vs. Sequential Offers

In many situations, it is difficult or impossible for the buyer to make simultaneous offers to the sellers. However, if the buyer can make simultaneous offers to all the sellers, there is an equilibrium in which the sellers agree in the first period because this game is of complete information. Therefore, we cannot discuss bargaining orders in such a setup.

4.8 Applications and Other Extensions

Besides land purchasing and the two other examples in the introduction, the model is also applicable to voting scenarios. For example, when a country wants to join a trade organization, it has to receive permission from all the respective existing members. The members have different attitudes toward the entry, and the member that prefers the entry least corresponds to the seller with largest size in our model. As a result, the applicant should start with the member who favors the entry most.

[25] See, for instance, Suh and Wen (2009) and Li (2010).

One of the most interesting extensions is to allow some players to hide some information, for example, the sellers' sizes, past offers or deal prices.[26] It would be interesting to examine how the bargaining order affects the players' incentive to reveal their private information. Moreover, the model has the potential for more applications with some other modifications. For example, it can be extended to the case where the buyer needs not purchase from all the sellers.[27] This modification accommodates the situation in which the developer can change the shape of the mall.[28] In addition, it also fits the voting situations where winning requires not only a minimum number of votes but also all the votes from voters with veto rights. Furthermore, it would be interesting to explore the situation where the bargaining order or the offers could be made confidential, as this paper only presents a game of complete information.[29]

5 Conclusion

This paper studies a non-cooperative and complete-information bargaining game with one buyer and multiple sellers who are heterogeneous in their sizes. The bargaining order in this game is endogenously determined, and three different cases are considered. First, if the bargaining game has a finite horizon, there is a unique equilibrium outcome in which the buyer always purchases from the smallest remaining seller. Second, consider the game with an infinite horizon. If the sellers' sizes are sufficiently different, there is also a unique equilibrium outcome in which the buyer bargains with the smallest remaining seller until an agreement is reached. If the sellers are similar to each other, the game still has an equilibrium with the same bargaining order, but there could be other equilibrium outcomes with different bargaining orders. Third, if the buyer has the option to commit to bargaining order, she could maximize her payoff by committing to the order of increasing size.

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[26] Noe and Wang (2004) examine a finite-horizon bargaining game in which the negotiation history could be kept secret.

[27] Chowdhury and Sengupta (2012) also consider this feature.

[28] For example, Yardley (2008) reports a boutique supermarket that was altered from its original design and built around a tiny house whose owner refused to sell.

[29] See also Noe and Wang (2004) and Chowdhury and Sengupta (2012).

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Appendices

A Proofs for Section 2

This appendix contains omitted proofs in Section 2. We first prove the results in the two-seller game. In particular, we prove Proposition 2 through Claims 1 to 14, then show Lemma 1. Then, we generalize these proofs to show Proposition 1 of the N-seller game.

Claim 1 *Consider the two-seller game with seller i and the buyer. The equilibrium price with any even horizon is larger than one with an odd horizon. That is, $p^1_{i,2t} > p^1_{i,2t'+1}$ for any natural numbers t and t'.*

Proof. Without loss of generality, consider a one-seller game with seller 1. The game is an alternating-offer bargaining game between the buyer and the seller. Consider such a game with $2t+2$ periods, $G(1, 2t+2)$. The buyer offers in the first period, and the alternating offering structure implies that seller 1 offers in the last period. Therefore, we have

$$\begin{aligned} p^1_{1,2t+2} &= H_{1,1} + \delta q^1_{1,2t+1}, \quad (25) \\ 1 - q^1_{1,2t+1} &= \delta(1 - p^1_{1,2t}). \quad (26) \end{aligned}$$

where (25) means that the buyer suggests a price of $p^1_{1,2t+2}$ such that the seller is indifferent between accepting and rejecting, and (26) means that the seller suggests a price of $q^1_{1,2t+1}$ such that the buyer is indifferent between accepting and rejecting.

Recall that $H_{1,1} = v_1(1-\delta)$, so we can express $p^1_{1,2t+2}$ in terms of $p^1_{1,2t}$ by solving for $q^1_{1,2t+1}$ from (26) and substituting it into (25). In particular, we obtain

$$p^1_{1,2t+2} - v_1 = (1 - v_1)(\delta - \delta^2) + \delta^2(p^1_{1,2t} - v_1). \quad (27)$$

If the one-seller game has only two periods, the equilibrium price is $p^1_{1,2} = v_1 + \delta(1 - v_1)$ by backward induction. We can rearrange this to get $p^1_{1,2} - v_1 = \delta(1 - v_1)$, then solve equation (27) recursively to obtain

$$p^1_{1,2t+2} - v_1 = (1 - v_1)(\delta - \delta^2 + \delta^3 - ... - \delta^{2t} + \delta^{2t+1}).$$

Hence,

$$p^1_{1,2t+2} = v_1 + \frac{\delta + \delta^{2t+2}}{1+\delta}(1 - v_1). \tag{28}$$

Similarly, the buyer offers in the final period of $G(1, 2t+1)$, therefore we have

$$\begin{aligned} p^1_{1,2t+1} &= H_{1,1} + \delta q^1_{1,2t}, \\ 1 - q^1_{1,2t} &= \delta(1 - p^1_{1,2t-1}), \end{aligned}$$

and

$$p^1_{1,2t+1} = v_1 + \frac{\delta - \delta^{2t+1}}{1+\delta}(1 - v_1). \tag{29}$$

Hence,

$$p^1_{1,2t+1} < v_1 + \frac{\delta}{1+\delta}(1 - v_1) < p^1_{1,2t'}$$

for any natural numbers t and t'. ■

Consider the two-seller game with T periods. We discuss even horizons and odd horizons separately. In particular, Claims 2 to 7 ensure that Proposition 2 is true for $T = 2t$, while Claims 10 to 14 ensure that Proposition 2 is true for $T = 2t + 1$. Since we only discuss the two-seller game before the proof of Proposition 1, we omit the term "two-seller" in the discussion before the proposition.

Let us briefly explain the proof for even horizons. We prove by induction. First, Claim 2 shows that the proposition is true if the horizon is 2 periods. If the horizon is 2 period, there is no delay because the first purchase is in period 1.

Second, suppose a game with horizon $2t$ has no delay and suppose Proposition 2 is true for the game. Then consider two possibilities in the game with horizon $2t + 2$. In the first possibility, there is no delay in the game with horizon $2t + 2$. We show in Claims 3 to 7 that bargaining with seller 1 first is dominated by bargaining with seller 2 first for the buyer. Therefore it must be seller 2 who sells if the horizon is $2t + 2$, and the proposition is true. In the second possibility, there is a delay in the game of a horizon $2t + 2$. Then, the first purchase happens only when there are $2t$ periods left, so the proposition is true for horizon $2t + 2$ by assumption. Furthermore, Claim 8 shows that, if a game has an even horizon that is longer than $2t + 2$, no agreement can be reached until there are $2t$ periods left. Therefore, seller 2 also sells first. To summarize, if there is delay with the horizon $2t+2$, the proposition is proved for any even horizon that is longer than $2t$; if there is no delay, the proposition is true for the horizon $2t + 2$.

Third, we proceed to consider the two possibilities for horizon $2t + 4$ as in the second step. Hence these considerations will complete the proof for even horizons.

Claim 2 *In the game with $T = 2$ periods, if the mall is profitable as in*

$$\delta(1 - v_1) - v_2 > 0, \tag{30}$$

there is a unique equilibrium. In the equilibrium, the buyer bargains with seller 2 *first, seller* 2 *sells in the first period at a price of* $p^1_{2,2} = v_2$*, and seller* 1 *sells at a price of* $p^1_{1,1} = v_1$ *in the second period.*

Proof. Backward induction is used to prove this claim. Let us first examine the final period. One possibility is that neither seller has agreed in this period, then the mall would not be built. The other possibility is that only seller i has not agreed in the last period. Since the buyer offers in the last period, she suggests to seller i a price of $p^1_{i,1} = v_i$, which the seller accepts.

Let us move to the first period. The buyer offers a price of $p^2_{i,2}$ to seller i in this period such that the seller is indifferent between accepting and rejecting. That is, $p^1_{i,2} = v_i$.

Therefore, if the buyer bargains with seller 2 first, her payoff is $\pi_{B,2} = \delta(1-p^1_{1,1}) - p^2_{2,2} = \delta(1-v_1) - v_2$, which is positive because of condition (30). In contrast, if the buyer bargains with seller 1 first, her payoff is $\pi'_{B,2} = \delta(1-v_2) - v_1$, which is lower than $\pi_{B,2}$ because $\delta < 1$. Hence the buyer prefers to bargain with seller 2 first. ■

Claim 3 *In a game with* $2t+2$ *periods, the first purchase cannot be in the second period.*

Proof. Suppose that there is no agreement in period 1, but seller i sells at a price of $q^2_{i,2t+1}$ in period 2. In period 1, the seller would accept any price above $p^2_{i,2t+1}$ where

$$p^2_{i,2t+2} = H_{i,1} + \delta q^2_{i,2t+1},$$

where the right hand side is seller i's payoff if he rejects $p^2_{i,2t+2}$. The buyer offers $p^2_{i,2t+2}$ in period 1 if it gives her a higher payoff than waiting one period. That is,

$$\delta(1-p^1_{j,2t+1}) - p^2_{i,2t+2} \geq \delta[\delta(1-p^1_{j,2t}) - q^2_{i,2t+1}]. \tag{31}$$

Substituting $p^2_{i,2t+2}$ into (31), we have

$$\delta(1-p^1_{j,2t+1}) - (H_{i,1} + \delta q^2_{i,2t+1}) \geq \delta[\delta(1-p^1_{j,2t}) - q^2_{i,2t+1}]$$

or,

$$\delta(1-p^1_{j,2t+1}) - v_i \geq \delta[\delta(1-p^1_{j,2t}) - v_i]$$

Since $p^1_{j,2t} > p^1_{j,2t+1}$ according to Claim 1, the inequality above is always true, so there is an agreement in the first period. This is a contradiction. ■

Claim 4 *Suppose that seller* 2 *sells in period* $2t_0$ *if the horizon is* $2t$*. If seller* 1 *sells in period 1 or 2 if the horizon is* $2t+2$*, the buyer's payoff is*

$$\pi'_{B,2t+2} = \delta(1-p^1_{2,2t+1}) - v_1 - \delta^{2t-2t_0+1}(p^1_{1,2t_0-1} - v_1).$$

Proof. If seller 1 sells in one of the first two periods of horizon $2t+2$, Claim 3 implies that he sells in the first period. Claim 3 implies that seller 2 in the game with horizon $2t$, must sell when there are $2t_0$ periods left. Moreover, Claim 2 implies that seller 2 sells if the horizon is 2, so $t_0 \geq 1$.

On one hand, suppose that seller 1 sells in period 1 in $G(B, 1, 2t+2)$ and $G(1, B, 2t+1)$, we can verify that

$$p^2_{1,2t+2} = H_{1,1} + \delta q^2_{1,2t+1}$$

and

$$q^2_{1,2t+1} = H_{1,2t-2t_0+2} + \delta^{2t-2t_0+2} p^2_{1,2t_0-1}.$$

Therefore, the buyer's payoff is

$$\begin{aligned} \pi'_{B,2t+2} &= \delta(1 - p^1_{2,2t+1}) - p^2_{1,2t+2} \\ &= \delta(1 - p^1_{2,2t+1}) - v_1 - \delta^{2t-2t_0+1}(p^1_{1,2t_0-1} - v_1). \end{aligned} \tag{32}$$

On the other hand, suppose that seller 1 sells in period 1 in $G(B, 1, 2t+2)$, but he does not sell in period 1 in $G(1, B, 2t+1)$. In the first period of $G(B, 1, 2t+2)$, if seller 1 rejects, he needs to wait $2t - 2t_0 + 1$ periods to sell his land. As a result, he accepts any price higher than $p^2_{1,2t+2} = H_{1,2t-2t_0+1} + \delta^{2t-2t_0+1} p^1_{1,2t_0-1}$. Therefore the buyer's payoff is

$$\begin{aligned} \pi'_{B,2t+2} &= \delta(1 - p^1_{2,2t+1}) - p^2_{1,2t+2} \\ &= \delta(1 - p^1_{2,2t+1}) - H_{1,2t-2t_0+1} - \delta^{2t-2t_0+1} p^1_{1,2t_0-1} \\ &= \delta(1 - p^1_{2,2t+1}) - v_1 - \delta^{2t-2t_0+1}(p^1_{1,2t_0-1} - v_1), \end{aligned}$$

which is the same as the one given by (32). ■

Claim 5 *If seller 2 agrees in period 1 or 2 if the horizon is $2t+2$, the buyer's payoff is*

$$\pi_B = \delta(1 - p^1_{1,2t+1}) - v_2 - \delta^{2t-2t_0}(p^2_{2,2t_0} - v_2).$$

Proof. Similar to Claim 4. ■

Claim 6 *Suppose seller 2 sells in period 1 if the horizon is any even number no greater than $2t$. In a game with a horizon of $2t$, seller 2's surplus is smaller than that of seller 1. That is,*

$$p^2_{2,2t} - v_2 \leq \delta(p^1_{1,2t-1} - v_1). \tag{33}$$

Proof. We prove this by induction. If $t = 1$, the inequality above becomes $v_2 - v_2 \leq \delta(v_1 - v_1)$, which is true. Suppose the claim is true for t, we will show that it holds for $t+1$. Suppose seller 2 sells in period 1 if the horizon is any even number no greater than $2t+2$. Since seller 2 sells in period 1 if the horizon is $2t+2$ or $2t+1$, we have

$$\begin{aligned} p^2_{2,2t+2} &= v_2(1 - \delta) + \delta q^2_{2,2t+1}, \\ \delta(1 - p^1_{1,2t}) - q^2_{2,2t+1} &= \delta(1 - p^1_{1,2t-1}) - p^2_{2,2t}. \end{aligned}$$

Therefore,

$$p^2_{2,2t+2} = v_2(1-\delta) + \delta\left(\delta(1-p^1_{1,2t}) - \delta(1-p^1_{1,2t-1}) + p^2_{2,2t}\right).$$

so,

$$\begin{aligned}
p^2_{2,2t+2} - v_2 &= v_2(1-\delta) + \delta\left(\delta(1-p^1_{1,2t}) - \delta(1-p^1_{1,2t-1}) + p^2_{2,2t}\right) - v_2 \\
&= -\delta^2 p^1_{1,2t} + \delta^2 p^1_{1,2t-1} + \delta(p^2_{2,2t} - v_2) \\
&\leq -\delta^2 p^1_{1,2t} + \delta^2 p^1_{1,2t-1} + \delta\delta(p^1_{1,2t-1} - v_1) \\
&< \delta\delta(p^1_{1,2t-1} - v_1) < \delta(p^1_{1,2t-1} - v_1),
\end{aligned}$$

where the first inequality comes from (33) and the second comes from Claim 1. ■

Claim 7 *Suppose that seller 1 sells in period 1 or 2 of $G(B,1,2t+2)$. Then, the buyer would be better off by bargaining with seller 2 first.*

Proof. Let us first characterize the necessary and sufficient conditions such that that seller 1 sells in period 1 of $G(1,B,2t+1)$. The buyer would accept any price lower than $q^2_{1,2t+1}$ such that

$$\delta(1-p^1_{2,2t}) - q^2_{1,2t+1} = \delta[\delta(1-p^1_{1},_{2t-1}) - p^2_{2,2t}]$$

and seller 1 would offer such a price if

$$q^2_{1,2t+1} \geq H_{1,2} + \delta p^1_{1,2t-1}.$$

The two conditions above imply that, seller 1 sells in period 1 of $G(B,1,2t+1)$ if and only if

$$\delta(1-p^1_{2,2t}) - \delta[\delta(1-p^1_{1},_{2t-1}) - p^2_{2,2t}] \geq H_{1,2} + \delta^2 p^1_{1,2t-1},$$

which is equivalent to

$$\delta(1-p^1_{2,2t}) - v_1 \geq \delta[\delta(1-v_1) - p^2_{2,2t}]. \tag{34}$$

Then, Claim 3 implies that if seller 1 sells in period 1 of $G(1,B,2t+1)$, he must sell in period 1 of $G(B,1,2t+2)$. Therefore, seller 1 sells in period 1 of $G(B,1,2t+2)$ and $G(1,B,2t+1)$ if and only if (34) is satisfied.

By the same argument, seller 1 does not agree in the first period of $G(1,B,2t+1)$ if (34) is not satisfied. In the first period of $G(B,1,2t+2)$, seller 1 accepts any price higher than $p^2_{1,2t+2}$ such that

$$p^2_{1,2t+2} = H_{1,3} + \delta^3 p^1_{1,2t-1},$$

and the buyer offers such a price if

$$\delta(1-p^1_{2,2t+1}) - p^2_{1,2t+2} \geq \delta^2[\delta(1-p^1_{1},_{2t-1}) - p^2_{2,2t}].$$

Substituting $p^2_{1,2t+2}$ into the inequality, we have

$$\delta(1-p^1_{2,2t+1}) - H_{1,3} - \delta^3 p^1_{1,2t-1} \geq \delta^2[\delta(1-p^1_{1},_{2t-1}) - p^2_{2,2t}].$$

Therefore, seller 1 agrees in period 1 of $G(B,1,2t+2)$ but not in period 1 of $G(1,B,2t+1)$ if and only if

$$
\begin{aligned}
\delta(1-p^1_{2,2t})-v_1 &< \delta[\delta(1-v_1)-p^2_{2,2t}], && (35)\\
\delta(1-p^1_{2,2t+1})-H_{1,3}-\delta^3 p^1_{1,2t-1} &\geq \delta^2[\delta(1-p^1_{1,2t-1})-p^2_{2,2t}]. && (36)
\end{aligned}
$$

Comparing conditions (34) to (36), we have that seller 1 sells in period 1 of $G(B,1,2t+2)$ if and only if (36) is satisfied.

Let us characterize the sufficient conditions for seller 2 to sell in one of the first two periods of $G(2,B,2t+1)$. Claim 3 implies that seller 2 sells in period 1 of $G(B,2,2t+2)$. Seller 2 sells in period 1 of $G(2,B,2t+1)$ if and only if one of the two following possibilities happen. First, seller 2 sells in period 1 of $G(B,2,2t+2)$ but not in period 1 of $G(2,B,2t+2)$. Second, seller 2 sells in period 1 of both $G(B,2,2t+2)$ and $G(2,B,2t+1)$. Similar to the derivation of condition (36), we only need to examine the condition for seller 2 to sell in period 1 of $G(B,2,2t+2)$ given that seller 2 sells in period 1 of $G(2,B,2t+1)$.

In period 1 of $G(2,B,2t+1)$, seller 2 accepts any price higher than

$$p^2_{2,2t+2}=H_{2,2}+\delta^2 p^2_{2,2t}.$$

The buyer offers such a price if

$$\delta(1-p^1_{1,2t+1})-p^2_{2,2t+2}\geq \delta^2[\delta(1-p^1_{1,2t-1})-p^2_{2,2t}].$$

Substituting $p^2_{2,2t+2}$ into the inequality above, we have

$$\delta(1-p^1_{1,2t+1})-H_{2,2}-\delta^2 p^2_{2,2t}\geq \delta^2[\delta(1-p^1_{1,2t-1})-p^2_{2,2t}]. \quad (37)$$

Therefore, seller 2 sells in the first two periods if (37) holds.

Recall that Claim 6 implies that $p^2_{2,2t}-v_2\leq\delta(p^1_{1,2t-1}-v_1)$, which implies

$$\delta(1-p^1_{1,2t+1})-H_{2,2}-\delta^2 p^2_{2,2t}>\delta(1-p^1_{2,2t+1})-H_{1,3}-\delta^3 p^1_{1,2t-1}.$$

Therefore, condition (36) implies (37). This means that if seller 1 sells in period 1 of $G(B,1,2t+2)$, seller 2 will sell in period 1 of $G(B,2,2t+2)$.

We can also verify that (33) implies $\pi_B>\pi'_B$, hence the buyer is better off by bargaining with seller 2 first if the horizon is $2t+2$. ■

Claim 8 *If there is no purchase in the first two periods of the game with $2t+2$ periods, there is no purchase in the first two periods of the game with $2t+4$ periods.*

Proof. Claim 7 implies that if there is a purchase in the first two periods, it must be from seller 2. Let us first characterize the condition for seller 2 not selling in the first two periods of $G(B, 2, 2t+2)$. Similar to Claim 7, the buyer rejects in period 1 of $G(2, B, 2t+1)$ if

$$\delta(1 - p^1_{1,2t}) - v_2 < \delta[\delta(1 - p^1_{1,2t-1}) - v_2]. \tag{38}$$

Given that the buyer rejects in period 1 of $G(2, B, 2t+1)$, seller 2 rejects in period 1 of $G(B, 2, 2t+2)$ if

$$\delta(1 - p^1_{1,2t+1}) - v_2 < \delta^2[\delta(1 - p^1_{1,2t-1}) - v_2]. \tag{39}$$

Suppose that there is no agreement in the first two periods of $G(2, B, 2t+1)$. Then, let us characterize the condition for no agreement in the first two periods of $G(B, 2, 2t+4)$. The buyer rejects in period 1 of $G(2, B, 2t+3)$ if

$$\delta(1 - p^1_{1,2t+2}) - v_2 < \delta^3[\delta(1 - p^1_{1,2t-1}) - v_2]. \tag{40}$$

Given that the buyer rejects in period 1 of $G(2, B, 2t+3)$, seller 2 rejects in period 1 of $G(B, 2, 2t+4)$ if

$$\delta(1 - p^1_{1,2t+3}) - v_2 < \delta^4[\delta(1 - p^1_{1,2t-1}) - v_2]. \tag{41}$$

Substituting (28) and (29) into (38) and (40), we can verify that the latter two are equivalent. Similarly, conditions (39) and (41) are also equivalent. This completes the proof of the claim.
■

So far we have proved the proposition for all even horizons. Let us consider the following claim for odd horizons.

Claim 9 *In the game with a horizon of $2t+1$ periods, if*

$$\delta(1 - p^1_{1,2t-1}) - v_2 \geq 0, \tag{42}$$

and

$$\delta(1 - p^1_{1,2t}) - v_2 > \delta[\delta(1 - p^1_{1,2t-1}) - v_2], \tag{43}$$

are satisfied, the mall is built. Moreover, the buyer bargains with seller 2 first, and seller 2 sells in period 1 of $G(B, 2, 2t+1)$ and $G(2, B, 2t)$. Otherwise, the mall is not built.

This claim above implies that the proposition is true for all odd horizons. We will prove this claim using Claims 10 to 16.

Claim 10 *In the game with $T = 3$ periods, there is a unique equilibrium. If*

$$\frac{1 - 2\delta}{1 - \delta}\delta(1 - v_1) - v_2 > 0$$

the mall is built, the buyer bargains with seller 2 first until an agreement is reached, **,** *seller 2 sells at price $p^2_{2,3} = v_2 + \delta[\delta(1 - v_1) - v_2]$ in period 1, seller 1 sells at price $p^1_{1,2} = v_1 + \delta(1 - v_1)$*

in period 2, and the buyer's payoff is

$$\pi_{B,3} = (1-\delta)\left[\frac{1-2\delta}{1-\delta}\delta(1-v_1) - v_2\right]. \tag{44}$$

If $\frac{1-2\delta}{1-\delta}\delta(1-v_1) - v_2 \leq 0$ *holds, the mall is not built.*

Proof. We prove this claim by backward induction. Let us first examine the final period. If neither seller has agreed in the last period, the mall is not built. Suppose that only seller 1 has not sold by the final period. If the buyer offers in the last period, she would suggest $p^1_{1,1} = v_1$ to seller 1. If seller 1 offers in the final period, he would suggest $q^1_{1,1} = 1$.

Let us move backwards to the second period. On the one hand, suppose that neither seller has agreed in this period. If seller 2 offers in period 2, he suggests $q^2_{2,2}$ such that the buyer is indifferent between accepting and rejecting. That is, $\delta(1-p^1_{1,1}) - q^2_{2,2} = 0$, so $q^2_{2,2} = \delta(1-v_1)$. On the other hand, suppose only seller 1 has not agreed. Then the buyer offers in period 2, and she offers $p^1_{1,2}$ to seller 1 such that he is indifferent between accepting and rejecting. That is, $p^1_{1,2} = H_{1,1} + \delta q^1_{1,1}$, where the right hand side is the harvest of period 2 and seller i's price in period 3. Therefore,

$$p^1_{1,2} = v_1 + \delta(1-v_1). \tag{45}$$

Finally, consider the first period. If the buyer bargains with seller 2 in the first period, she offers $p^2_{2,3}$ such that seller 2 is indifferent between accepting and rejecting. That is, $p^2_{2,3} = H_{2,1} + \delta q^2_{2,2}$. Substituting $H_{2,1}$ and $q^2_{2,2}$ into the equation above, we have $p^2_{2,3} = v_2 + \delta[\delta(1-v_1) - v_2]$. As a result, if the buyer bargains with seller 2 in the first period, her payoff is $\pi_{B,3} = \delta(1-p^1_{1,2}) - p^2_{2,3} = \delta(1-2\delta)(1-v_1) - v_2(1-\delta)$.

In contrast, if the buyer bargains with seller 1 in the first period, her payoff is $\pi'_{B,3} = \delta(1-p^1_{2,2}) - p^2_{1,3}$. Since $\delta < 1$ and $1-2\delta < 1-\delta$, we have $\delta(1-2\delta) < 1-\delta$, which implies that $\pi_{B,3} > \pi'_{B,3}$, therefore the buyer chooses seller 2 in the first period. ■

Claim 10 shows that Claim 9 is true for the horizon of 3 periods. Assuming that Claim 9 is true for a horizon of $2t-1$ periods, we prove that Claim 9 is true for horizon $2t+1$ in two cases. Claims 11 to 15 consider the first case in which the mall is built if the horizon is $2t-1$. Claim 16 considers the second case in which the mall is not built if the horizon is $2t-1$.

Claim 11 *If the mall is built in the game with horizon* $2t-1$, *seller* 2 *sells in period 1 of* $G(B, 2, 2t+1)$ *and* $G(2, B, 2t)$.

Proof. Let us characterize the conditions for seller 2 to sell in period 1 of $G(B, 2, 2t+1)$ and $G(2, B, 2t)$. Similar to the proof of Claim 7, seller 2 sells in period 1 of $G(2, B, 2t)$ if

$$\delta(1-p^1_{1,2t-1}) - v_2 \geq \delta[\delta(1-p^1_{1,2t-2}) - v_2], \tag{46}$$

which is always true because $p^1_{1,2t-1} < p^1_{1,2t-2}$. Seller 2 sells in period 1 of $G(B, 2, 2t+1)$ if (43) holds. Therefore seller 2 agrees in the first period of $G(B, 2, 2t+1)$ and $G(2, B, 2t)$ if

(43) is satisfied. We can verify that (43) is equivalent to

$$\delta(1-v_1)\alpha - v_2 \geq 0, \tag{47}$$

where

$$\alpha = \left[1 - \frac{\delta+\delta^{2t}}{1+\delta} - \delta\left(1 - \frac{\delta-\delta^{2t-1}}{1+\delta}\right)\right] / (1-\delta).$$

Suppose that the mall is built in the game with horizon $2t-1$, then Claim 9 implies that

$$\delta(1-p^1_{1,2t-2}) - v_2 > \delta[\delta(1-p^1_{1,2t-3}) - v_2], \tag{48}$$

which is equivalent to

$$\delta(1-v_1)\alpha' - v_2 > 0,$$

with

$$\alpha' = \left[1 - \frac{\delta+\delta^{2t-2}}{1+\delta} - \delta\left(1 - \frac{\delta-\delta^{2t-3}}{1+\delta}\right)\right] / (1-\delta).$$

Since $\alpha' < \alpha$, we have

$$\delta(1-v_1)\alpha - v_2 > \delta(1-v_1)\alpha - v_2.$$

Therefore (48) implies (43), and seller 2 sells in period 1 of $G(B,2,2t+1)$ and $G(2,B,2t)$. ■

Claim 12 *Suppose that the mall is built in the game with horizon* $2t-1$*. If seller* 2 *sells in period 1 of* $G(B,2,2t+1)$ *and* $G(2,B,2t)$*, the buyer's payoff is*

$$\pi_{B,2t+1} = \delta(1-p^1_{2,2t}) - v_1(1-\delta) - \delta\left(\delta(1-p^1_{2,2t-1}) - \delta\pi_{B,2t-1}\right).$$

Proof. In the game $G(2,B,2t)$, seller 2 offers a price of $q^2_{2,2t}$ such that the buyer is indifferent between accepting and rejecting. That is,

$$\delta(1-p^1_{1,2t-1}) - q^2_{2,2t} = \delta\pi_{B,2t-1}. \tag{49}$$

In the game $G(B,2,2t+1)$, the buyer offers a price of $p^2_{2,2t+1}$ such that seller 2 is indifferent between accepting and rejecting. That is,

$$p^2_{2,2t+1} = H_{2,1} + \delta q^2_{2,2t}. \tag{50}$$

The buyer's payoff in $G(B,2,2t+1)$ is

$$\begin{aligned}
\pi_{B,2t+1} &= \delta(1-p^1_{1,2t}) - p^2_{2,2t+1} \\
&= \delta(1-p^1_{1,2t}) - v_2(1-\delta) - \delta q^2_{2,2t} \\
&= \delta(1-p^1_{1,2t}) - v_2(1-\delta) - \delta\left(\delta(1-p^1_{1,2t-1}) - \delta\pi_{B,2t-1}\right),
\end{aligned} \tag{51}$$

where the second equality comes from (50) and the third equality comes from (49). ■

Claim 13 *Suppose that the mall is built in the game with horizon $2t-1$. If seller 1 sells in period 1 of $G(B,1,2t+1)$ and $G(1,B,2t)$, the buyer is better off by bargaining with seller 2 first.*

Proof. The proof is similar to the one for Claim 12. In the game $G(1,B,2t)$, seller 1 offers a price of $q^2_{1,2t}$ such that

$$\delta(1-p^1_{2,2t-1})-q^2_{1,2t}=\delta\pi_{B,2t-1}.$$

In the game $G(B,1,2t+1)$, the buyer offers a price of $p^2_{1,2t+1}$ such that

$$p^2_{1,2t+1}=H_{1,1}+\delta q^2_{1,2t}.$$

Then the buyer's payoff in $G(B,1,2t+1)$ is

$$\begin{aligned}\pi'_{B,2t+1} &= \delta(1-p^1_{2,2t})-p^2_{1,2t+1}\\ &= \delta(1-p^1_{2,2t})-v_1(1-\delta)-\delta q^2_{1,2t}\\ &= \delta(1-p^1_{2,2t})-v_1(1-\delta)-\delta\left(\delta(1-p^1_{2,2t-1})-\delta\pi_{B,2t-1}\right).\end{aligned}\tag{52}$$

Suppose the buyer bargains with seller 2 instead. Then, Claim 11 implies that seller 2 sells in period 1 of $G(B,2,2t+1)$ and $G(2,B,2t)$. Therefore, Claim 12 implies that the buyer's payoff $\pi_{B,2t+1}$ is given by (51). In contrast, if the buyer bargains with seller 1 first, her payoff $\pi'_{B,2t+1}$ is given by (52). By the same argument to show (48) implies (43), we have $\pi_{B,2t+1}>\pi'_{B,2t+1}$. ■

Claim 14 *Suppose that the mall is built in the game with horizon $2t-1$. If seller 1 does not sell until period 2 of $G(B,1,2t+1)$, the buyer is better off by bargaining with seller 2 first.*

Proof. Suppose the buyer bargains with seller 2 instead, then Claim 11 implies that seller 2 sells in period 1 of $G(B,2,2t+1)$ and $G(2,B,2t)$.

If seller 1 does not sell until period 2 of $G(B,1,2t+1)$, seller 2 sells in period 1 of $G(B,2,2t+1)$ and $G(2,B,2t)$. In $G(2,B,2t)$, seller 2 offers a price such that the buyer is indifferent between accepting and rejecting, so the buyer's payoff is $\pi_{B,2t}=\delta\pi_{B,2t-1}$. Then, conditions (43) and (51) imply that $\pi_{B,2t+1}>\delta^2\pi_{B,2t-1}$. ■

Claim 15 *Suppose the mall is built in the game with horizon $2t-1$. If seller 1 sells in period 1 of $G(B,1,2t+1)$ but not in period 1 of $G(1,B,2t)$, the buyer is better off by bargaining with seller 2 first.*

Proof. In period 1 of $G(1,B,2t)$, the buyer accepts any price lower than $q^2_{1,2t}$ such that

$$\delta(1-p^1_{2,2t-1})-q^2_{1,2t}=\delta[\delta(1-p^1_{1,2t-2})-p^2_{2,2t-1}].\tag{53}$$

Seller 2 offers $q^2_{1,2t}$ if

$$q^2_{1,2t}\geq H_{1,1}+\delta p^2_{1,2t-2}.\tag{54}$$

Solving for $q^2_{i,2t+2}$ from (53) and substituting it into (54), we have

$$\delta(1 - p^1_{2,2t-1}) - v_1 \geq \delta[\delta(1 - v_1) - p^2_{2,2t-1}]. \tag{55}$$

Since seller 1 does not sell in period 1 of $G(1, B, 2t)$, condition (55) must be violated. That is,

$$\delta(1 - p^1_{2,2t-1}) - v_1 < \delta[\delta(1 - v_1) - p^2_{2,2t-1}]. \tag{56}$$

Suppose the buyer bargains with seller 2 instead, Claim 11 implies that seller 2 sells in period 1 of $G(B, 2, 2t + 1)$ and $G(2, B, 2t)$.

If seller 1 sells in period 1 of $G(B, 1, 2t+1)$ but not in period 1 of $G(1, B, 2t)$, the buyer's payoff in $G(B, 1, 2t + 1)$ is

$$\begin{aligned} \pi''_{B,2t+1} &= \delta(1 - p^1_{2,2t}) - p^2_{1,2t+1} \\ &= \delta(1 - p^1_{2,2t}) - H_{1,3} - \delta^3 p^1_{1,2t-2}. \end{aligned}$$

According to (51), the buyer's payoff is

$$\begin{aligned} \pi_{B,2t+1} &= \delta(1 - p^1_{1,2t}) - v_2(1 - \delta) \\ &\quad -\delta\left(\delta(1 - p^1_{1,2t-1}) - \delta(\delta(1 - p^1_{1,2t-2}) - p^2_{2,2t-1})\right) \end{aligned}$$

Therefore,

$$\begin{aligned} &\pi_{B,2t+1} - \pi''_{B,2t+1} \\ = \;&\delta(1 - p^1_{1,2t}) - v_2 - \left[\delta(1 - p^1_{2,2t}) - v_1\right] \\ &-\delta\left[\delta(1 - p^1_{1,2t-1}) - v_2\right] + \delta^2(\delta(1 - v_1) - p^2_{2,2t-1}) \\ \geq \;&\delta(1 - p^1_{1,2t}) - v_2 - \left[\delta(1 - p^1_{2,2t}) - v_1\right] > 0, \end{aligned}$$

where the first inequality comes from (56) and the last inequality comes from a similar argument to prove that (48) implies (43). ■

Claim 16 *Suppose that the mall is not built if the horizon is $2t - 1$. If (42) and (43) are satisfied, seller 2 sells in period 1. Otherwise the mall is not built.*

Proof. In period 1 of $G(2, B, 2t)$, seller 2 offers $q^2_{2,2t}$ such that the buyer is indifferent between accepting and rejecting. That is,

$$\delta(1 - p^1_{1,2t-1}) - q^2_{2,2t} = 0. \tag{57}$$

Seller 2 offers $q^2_{2,2t}$ if

$$q^2_{2,2t} \geq v_2.$$

Solving $q^2_{2,2t}$ from (57) and substituting it into the inequality above, we get (42). As a result, seller 2 sells in period 1 of $G(2, B, 2t)$ if (42) is satisfied.

Given the mall is built in $G(2, B, 2t)$, let us consider the game $G(B, 2, 2t+1)$. In period 1, seller 2 accepts any price no lower than $p^2_{2,2t+1}$ such that

$$p^2_{2,2t+1} = H_{2,1} + \delta q^2_{2,2t}$$

and the buyer offers $p^2_{2,2t+1}$ if

$$\delta(1 - p^1_{1,2t}) - p^2_{2,2t+1} \geq \delta \left[\delta(1 - p^1_{1,2t-1}) - q^2_{2,2t}\right]. \tag{58}$$

Substituting $p^2_{2,2t+1}$ into the inequality above, we get (43) with a weak inequality. Recall that the buyer does not initiate the bargaining unless her payoff is going to be positive, so the weak inequality should be strict. Hence, seller 2 sells in period 1 of $G(B, 2, 2t+1)$ and $G(2, B, 2t)$ if (42) and (43) are satisfied. By the above analysis, these conditions are also necessary for seller 2 to sell in period 1 of $G(B, 2, 2t+1)$ and $G(2, B, 2t)$.

Suppose that seller 1 does not sell until period 2 of $G(B, 1, 2t+1)$. We can verify that the buyer's payoff is zero. Then, the buyer would not initiate the bargaining and the mall is not built. Therefore, if the mall is built in $G(B, 1, 2t+1)$, seller 1 must sell in period 1.

Suppose that seller 1 to sell in period 1 of $G(B, 1, 2t+1)$ and $G(1, B, 2t)$. Similar to the poof of Claim 13, the buyer would be better off by bargaining with seller 2 first.

Suppose that seller 1 sells in period 1 of $G(B, 1, 2t+1)$ but not in period 1 of $G(1, B, 2t)$, the buyer would be better off by bargaining with seller 2 first. This can be proved in a similar manner to the proof of Claim 15. ■

Proof of Proposition 2. So far, we have shown that selling time (t_1, t_2) – the periods in which the sellers sell their land – is unique. As in Claims 2 and 10, the equilibrium prices are also unique by backward induction as long as the selling time is given. We have completed the proof for Proposition 2. ■

Proof of Lemma 1. Suppose that seller 2 sells in the first period if the horizon is T. By a similar argument to the proof of Claim 7, the buyer agrees in the first period of $G(2, B, T+1)$ if and only if

$$\delta(1 - p^1_{1,T}) - v_2 \geq \delta[\delta(1 - p^1_{1,T-1}) - v_2]. \tag{59}$$

Moreover, seller 2 agrees in the first period of $G(B, 2, T+2)$ if and only if

$$\delta(1 - p^1_{1,T+1}) - v_2 \geq \delta[\delta(1 - p^1_{1,T}) - v_2]. \tag{60}$$

If $\frac{1}{1+\delta}\delta(1-v_1) - v_2 < 0$ is satisfied, there exists $t_0 > 0$ such that neither of the two inequalities hold if $T \geq t_0$. Therefore, if the buyer keeps bargaining with seller 2 first, there is a delay of at least $T - t_0$ periods. Similarly, if the buyer chooses to bargain with seller 1 first, there is also a delay of at least $T - t_0$ periods. Hence, if T goes to infinity, the length of delay also goes to infinity. Hence, if $\frac{1}{1+\delta}\delta(1 - v_1) - v_2 < 0$, the surplus for every player converges to zero.

On the other hand, suppose that (4) is satisfied. Then, there exists $t_1 > 0$ such that (59) and (60) are satisfied for $T \geq t_1$. Suppose no agreement is reached in the first period for any even horizon longer than t_1. If the horizon is long enough, the buyer's payoff converges to zero, and the seller i's payoff converges to v_i. Consider a deviation in which the buyer suggests a price of $v_2 + \varepsilon$ to seller 2 with some small $\varepsilon > 0$. Seller 2 would accept it as it results in a payoff higher than v_2. The buyer's payoff converges to

$$\delta \frac{1}{1+\delta}(1 - v_1) - v_2 - \varepsilon > 0$$

because of (4). Therefore, there exists an even horizon $T_1 > t_1$ such that seller 2 sells in the first period of $G(B, 2, T)$ and $G(2, B, T + 1)$. Similar to Claims 13 to 15, the buyer would not bargain with seller 1 first because it would result in a lower payoff for her.

Consider a horizon T longer than T_1 and T_1'. The above analysis implies that the buyer offers to seller 2 in period 1 such that he is indifferent between accepting and rejecting. That is,

$$p^2_{2,T} = H_{2,1} + \delta q^2_{2,T-1}.$$

If no agreement is reached in period 1, seller 2 offers $q^2_{2,T-1}$ in period 2 such that the buyer is indifferent between accepting and rejecting. That is,

$$\delta(1 - p^1_{1,T-2}) - p^2_{2,T-1} = \delta[\delta(1 - p^1_{1,T-3}) - p^2_{2,T-2}].$$

As T converges to infinity, the two above equations become

$$\begin{aligned} \lim_{T\to\infty} p^2_{2,T} &= H_{2,1} + \delta \lim_{T\to\infty} q^2_{2,T-1} \\ \delta(1 - \lim_{T\to\infty} p^1_{1,T-2}) - \lim_{T\to\infty} q^2_{2,T-1} &= \delta[\delta(1 - \lim_{T\to\infty} p^1_{1,T-3}) - \lim_{T\to\infty} p^2_{2,T-2}]. \end{aligned}$$

It is easy to see that $\lim_{T\to\infty} p^2_{2,T} = \lim_{T\to\infty} p^2_{2,T-2}$ and $\lim_{T\to\infty} p^1_{1,T-2} = v_1 + \frac{\delta}{1+\delta}(1 - v_1)$, so we can solve for the limit equilibrium prices and obtain (5) to (7). ■

Proof of Proposition 1. The proof of Lemma 1 applies analogously to the N-seller game. As a result, we only sketch the proof for $N = 3$ here. First, Lemma 1 shows that Proposition 1 is true for the two-player game. Suppose the proposition is true for the $(N-1)$-seller game, we can show that the proposition is also true for the N-seller game.

In particular, if $\sum_{i=1}^{N} \left(\frac{\delta}{1+\delta}\right)^{N-i} v_i > \left(\frac{\delta}{1+\delta}\right)^{N-1}$ is satisfied and if T goes to infinity, the length of delay goes to infinity. Hence, the surplus for every player converges to zero. If (3) is satisfied and if T is large enough, seller N sells in period 1 if the buyer keeps bargaining with him first. Similar to Claims 13 to 15, we can show that the buyer would not bargain with other seller first because it would result in a lower payoff for her.

Let $\mathbf{v} = (v_1, v_2, ..., v_N)$ be the vector of all sellers' values and $\mathbf{v}_{-N} = (v_1, v_2, ..., v_{N-1})$ be the vector of values of all the sellers except N. Denote $\pi^{N-1}_{B,T}(\mathbf{v}_{-N})$ as the buyer's payoff in the game with a horizon of T periods and sellers $1, 2, ..., N-1$. Then, under (3), the equilibrium prices as T goes to infinity satisfy:

$$\begin{aligned} \lim_{T\to\infty} p^N_{N,T} &= H_{N,1} + \delta \lim_{T\to\infty} q^N_{N,T-1}, & (61)\\ \delta \lim_{T\to\infty} \pi^{N-1}_{B,T}(\mathbf{v}_{-N}) - \lim_{T\to\infty} q^N_{N,T-1} &= \delta \lim_{T\to\infty} \pi^N_{B,T}(\mathbf{v}). & (62)\end{aligned}$$

The buyer's equilibrium payoff is

$$\lim_{T\to\infty} \pi^N_{B,T}(\mathbf{v}) = \delta \lim_{T\to\infty} \pi^{N-1}_{B,T}(\mathbf{v}_{-N}) - \lim_{T\to\infty} p^N_{N,T}. \tag{63}$$

Hence, solving $\lim_{T\to\infty} p^N_{N,T}$ and $\lim_{T\to\infty} q^N_{N,T-1}$ from (61) and (62) and substituting them into (63), we have

$$\lim_{T\to\infty} \pi^N_{B,T}(\mathbf{v}) = \frac{1}{1+\delta}[\delta \lim_{T\to\infty} \pi^{N-1}_{B,T}(\mathbf{v}_{-N}) - v_N].$$

Then we can solve $\lim_{T\to\infty} \pi^N_{B,T}(\mathbf{v})$ recursively using the equation above. In addition, we can verify that $\lim_{T\to\infty} \pi^N_{B,T}(\mathbf{v}) > 0$ is equivalent to (3). According to (63), we have

$$\lim_{T\to\infty} \pi^N_{B,T}(\mathbf{v}) = \delta \lim_{T\to\infty} \pi^{N-1}_{B,T}(\mathbf{v}_{-N}) - \lim_{T\to\infty} p^N_{N,T} < \lim_{T\to\infty} \pi^{N-1}_{B,T}(\mathbf{v}_{-N}),$$

which combined with (62) implies $\lim_{T\to\infty} q^N_{N,T-1} > 0$. Therefore, (61) implies $\lim_{T\to\infty} p^N_{N,T} > H_{N,1}$. Hence, the limit surplus for player N under (3) is $\lim_{T\to\infty} p^N_{N,T} - H_{N,1} > 0$ as T goes to infinity. ■

B Proofs for Section 3.2

This appendix contains omitted proofs in Section 3.2. In particular, Lemma 7 and Proposition 4 discuss the two-seller game, then Lemma 8 and Proposition 3 generalize the analysis to the N-seller game.

Lemma 7 *Perpetual disagreement cannot be an equilibrium outcome.*

Proof. It is easy to see that $\pi_B = 0$ under perpetual disagreement.

If the buyer first purchases from seller 1, she would have negative payoff. Therefore, seller 2 is the first seller and sells in period t_2 for a price of p^2_2, while seller 1 sells in the next period at price p^1_1. In order to maintain a balanced budget, the total payments to all the players should equal the total value of the mall,

$$p^2_2 + \delta p^1_1 + \delta \pi_B = \delta,$$

which implies

$$p^2_2 = \delta - \delta p^1_1 - \delta \pi_B. \tag{64}$$

Seller 2's payoff is given by

$$
\begin{aligned}
\pi_2 &= H_{2,t_2-1} + \delta^{t_2-1} p_2^2 \\
&= H_{2,t_2-1} + \delta^{t_2-1}\left(\delta - \delta p_1^1 - \delta \pi_B\right),
\end{aligned}
$$

where the second inequality comes from (64). π_2 is a function of t_2 and π_B, with an upper bound of

$$
\begin{aligned}
\overline{\pi}_2 &\equiv \max_{t_2 \geq 2, \pi_B \geq 0} \left[H_{2,t_2-1} + \delta^{t_2-1}\left(\delta - \delta p_1^1 - \delta \pi_B\right)\right] \\
&= \delta - \delta p_1^1.
\end{aligned}
$$

When seller 2 receives an offer, his payoff also faces the same upper bound. As a result, if seller 2 rejects the buyer's offer, he receives one period of harvest in the current period and at most $\overline{\pi}_2$ in the next period. Since $H_{2,1} + \delta\overline{\pi}_2 < \overline{\pi}_2$, there exists $\varepsilon > 0$ such that

$$
H_{2,1} + \delta\overline{\pi}_2 + \varepsilon < \overline{\pi}_2.
$$

If the buyer offers to seller 2 the price of $H_{2,1} + \delta\overline{\pi}_2 + \varepsilon$ in period 1, seller 2 would accept it as he would receive at most $H_{2,1} + \delta\overline{\pi}_2$ otherwise. Then, the buyer's payoff π_B satisfies the following budget balancing condition:

$$
\left(H_{2,1} + \delta\overline{\pi}_2 + \varepsilon\right) + \delta p_1^1 + \delta \pi_B = \delta
$$

so,

$$
\begin{aligned}
\pi_B &= -\left(H_{2,1} + \delta\overline{\pi}_2 + \varepsilon\right) - p_1^1 + 1 \\
&> -\left(H_{2,1} + \delta\overline{\pi}_2\right)/\delta - p_1^1 + 1 = 0
\end{aligned}
$$

The buyer could always guarantee himself a positive payoff by offering $H_{2,1} + \delta\overline{\pi}_2 + \varepsilon$ to seller 2 in the first period. Hence, permanent disagreement cannot be an equilibrium outcome. ■

Proof of Proposition 4. Lemma 7 implies that there is always a first seller. Suppose the first seller sells in period t of $\Gamma(B, 2)$ and that $t > 1$. As discussed after Lemma 3, the buyer would receive a negative payoff if seller 1 sells first, so the first seller must be seller 2. Let m_B and M_B be the infimum and supremum of equilibrium payoffs of the buyer, where m_B, $M_B > 0$.

Let (π_1, π_2, π_B) be a payoff vector, and $U(j, j')$ denote the set of all equilibrium payoff vectors of subgame $\Gamma(j, j')$.

Suppose seller 2 sells immediately at price p_2 in subgame $\Gamma(B, 2)$, then seller 2 and the buyer's payoffs are

$$
\begin{aligned}
\pi_B &= \delta - \delta p_1^1 - p_2 \\
\pi_2 &= p_2,
\end{aligned}
$$

which imply

$$\pi_2 + \pi_B = \delta - \delta p_1^1 \equiv \pi_{-1}(B,2).$$

Similarly, if seller 2 sells in the first period of subgame $\Gamma(2,B)$, we have

$$\pi_2 + \pi_B = \pi_{-1}(B,2).$$

If seller 2 sells at price p_2 in the third period of $\Gamma(B,1)$, the payoffs for seller 2 and the buyer are

$$\begin{aligned}\pi_B &= \delta^3 - \delta^3 p_1^1 - \delta^2 p_2,\\ \pi_2 &= H_{2,2} + \delta^2 p_2,\end{aligned}$$

which imply

$$\pi_2 + \pi_B = \delta^3 - \delta^3 p_1^1 + H_{2,2} \equiv \pi_{-1}(B,1).$$

Similarly, if seller 2 sells in the second period of $\Gamma(1,B)$, we have

$$\pi_2 + \pi_B = \delta^2 - \delta^2 p_1^1 + H_{2,1} \equiv \pi_{-1}(1,B).$$

Let m_{B2} and M_{B2} be the infimum and supremum of π_2 in $U(B,2)$ and m_{2B} and M_{2B} be the infimum and supremum of π_2 in $U(2,B)$. In $\Gamma(B,2)$, we have

$$M_{B2} = H_{2,1} + \delta M_{2B}, \tag{65}$$

$$m_{B2} = H_{2,1} + \delta m_{2B}. \tag{66}$$

In $\Gamma(2,B)$, we have

$$\pi_{-1}(2,B) - M_{2B} = \delta M_B, \tag{67}$$

$$\pi_{-1}(2,B) - m_{2B} = \delta m_B. \tag{68}$$

Since there would be no agreement in the first two periods of subgame $\Gamma(B,1)$, the infimum and supremum of π_B in the subgame are $\delta^2 M_B$ and $\delta^2 m_B$.

Since $\delta < 1$ and m_B, $M_B > 0$, the infimum and supremum of π_B for the whole game must satisfy

$$M_B = \pi_{-1}(B,2) - m_{B2}, \tag{69}$$

$$m_B = \pi_{-1}(B,2) - M_{B2}. \tag{70}$$

Since there is a unique solution to equations (65) to (70), we have

$$\begin{aligned}M_{B2} &= m_{B2},\\ M_{2B} &= m_{2B},\\ M_B &= m_B.\end{aligned}$$

Suppose that there is an equilibrium with delay, then the equilibrium payoff of the buyer must be lower than that in an equilibrium without delay. Thus we have a contradiction as we would have $m_B < M_B$. Hence, there is a unique element in the sets $U(B,2)$, $U(2,B)$, $U(B,1)$ and $U(1,B)$. ■

Now consider the N-seller game without commitment. Lemma 8 below and Proposition 3 extends our analysis of the two-seller game to the N-seller case. Let π_B^{N-1*} be the unique equilibrium payoff in the $(N-1)$-seller game. As in Lemma 3, let $\left(p_N^N, q_N^N\right)$ be the solution to the equations

$$p_N = H_{N,1} + \delta q_N, \tag{71}$$

$$\delta\pi_B^{N-1*} - q_N = \delta\left(\delta\pi_B^{N-1*} - p_N\right). \tag{72}$$

There is an equilibrium where the buyer suggests a price p_N^N to seller N and accepts a price no more than q_N^N from seller N; and seller N suggests price q_N^N and accepts a price no less than p_N^N. The corresponding equilibrium payoff for the buyer is

$$\pi_B^{N*} = \delta\pi_B^{N-1*} - p_N^N$$

and it is easy to show by induction that

$$\pi_B^{N*} = \frac{1}{1+\delta}\left(\left(\frac{\delta}{\delta+1}\right)^{N-1} - \sum_{i=1}^{N}\left(\frac{\delta}{1+\delta}\right)^{N-i} v_i\right). \tag{73}$$

Equation (73) also defines a function $\pi_B^{N*}(v_1, ..., v_N)$ for any N, so $\pi_B^{N-1*}(\mathbf{v}_{-i})$ denotes the equilibrium payoff in the $(N-1)$-seller game without seller i, where $\mathbf{v}_{-i} = (v_1, ..., v_{i-1}, v_{i+1}, ..., v_N)$. It is easy to see from (73) that π_B^{n*} is a linear function of $v_1, ..., v_n$, and v_i's coefficient is smaller than v_{i+1}.

Lemma 8 *For every $i \in \{1, ..., N-1\}$, let $\left(p_i^N, q_{Bi}^N\right)$ be the solution to the equations*

$$p_i = H_{i,N+1} + \delta^{N+1} p_i^i, \tag{74}$$

$$\delta\pi_B^{N-1^*}(\mathbf{v}_{-i}) - q_{Bi} = \delta\pi_B^{N*}. \tag{75}$$

If (9) for $n = 2, ..., N$ and (8) are satisfied, given any (q_i^N, p_{Bi}^N) such that $q_i^N > q_{Bi}^N$ and $p_{Bi}^N < q_i^N$, the following strategies constitute an equilibrium for the game $\Gamma(B,N)$:

i) seller N suggests a price of q_N^N and accepts a price no less than p_N^N,

ii) seller i suggests a price of q_i^N and accepts a price no less than p_i^N for $i = 1, \cdots, N-1$,

iii) the buyer bargains with seller N before the first agreement; suggests a price of p_N^N to seller N and a price p_{Bi}^i to seller $i = 1, \cdots, N-1$; and accepts a price no more than q_N^N from seller N and a price no more than q_{Bi}^N from seller $i = 1, \cdots, N-1$.

Proof. Induction on the number of sellers is used. Suppose Lemma 8 is true for $N = k$. By the same backward induction analysis as in Lemma 3, the above lemma is also true for

$N = k+1$. Therefore, only the arguments relating to the interpretations are given below.

The conditions in this lemma also have similar interpretations to those in Lemma 3. For any $i < N$, (74) means that seller i is indifferent between accepting and rejecting the buyer's offer p_i^N, and (75) ensures that the buyer is indifferent between accepting and rejecting seller i's offer q_{Bi}^N.

The sellers sell in the order of increasing size in the first N periods. If seller $N-1$ and seller N exchange their selling periods, the buyer's payoff is $\pi_B^{N*}(v_1, ..., v_{N-2}, v_N, v_{N-1})$, and condition (9) for $n = N$ is equivalent to

$$\pi_B^{N*}\left(v_1, ..., v_{N-2}, v_N, v_{N-1}\right) < 0.$$

Since v_i's coefficient is smaller than v_{i+1}'s in π_B^{N*} for all i, condition (9) also implies that the buyer receives a negative payoff if any seller other than N sells first. Moreover, (9) holds for $n = 2, ..., N$, so the smallest remaining seller has to be the first to sell, otherwise the buyer receives a negative payoff. ■

Proof of Proposition 3: Analogous to Proposition 4, the equilibrium outcome implied by Lemma 8 is also unique in $\hat{\Gamma}\left(B, N\right)$. As in the two-seller game, if the buyer chooses a seller other than N to bargain with first, the resulting subgame is a proper subgame of $\Gamma\left(B, N\right)$. This means there is no agreement in the first two periods, and the buyer chooses seller N to bargain with in the third period and an agreement is reached immediately. As a result, if the buyer bargains with any other seller first, all the sale prices remain the same, but there would be two periods of delay. Hence the buyer bargains with the smallest remaining seller. ■

C Proofs for Section 3.3

This appendix contains omitted proofs in Section 3.3. In particular, we first prove Proposition 5 in the two-seller game through Claims 17 to 21. Then, we generalize the proof to show Proposition 5. Let $\bar{\pi}_B$ be the supremum of the buyer's equilibrium payoffs. This proof consists of five claims.

Claim 17 *If the buyer's payoff is π_B in an equilibrium, the buyer's payoff is at least π_B in another equilibrium with an agreement in the first period.*

Proof. Suppose the buyer's payoff is π_B in the equilibrium E, where the bargaining order is $i_1, i_2, ...$, and the first agreement is reached in period $t > 1$. Then, E induces an equilibrium, E_t for the subgame $\hat{\Gamma}\left(B, (i_t, i_{t+1}, ...)\right)$. Note that the buyer does not choose another bargaining order in the subgame $\hat{\Gamma}\left(B, (i_t, i_{t+1}, ...)\right)$, but the order chosen in the first period also specifies the sequence of sellers, $i_t, i_{t+1}, ...$, in this subgame.

If there is no agreement in E, the buyer receives zero payoff in both E and E_t. If there is an agreement in E, the sellers sell at the same prices in E_t as in E but $t-1$ periods earlier, therefore the buyer receives a higher payoff than in E. Hence the buyer's payoff in E_t is at least π_B. As a result, it is another equilibrium where the buyer chooses order $i_t, i_{t+1}, ...$ and every player follows the strategies in E_t. In this equilibrium, the buyer's payoff is at least π_B and an agreement is reached in period 1. ■

Claim 18 *The payoff $\bar{\pi}_B$ can be approached by the buyer's equilibrium payoffs in the subgames with either order* $1, 1, ...$ *or order* $2, 2, ...$

Proof. Since $\bar{\pi}_B$ can be approached by a sequence of identical payoffs, this claim does not exclude the possibility of a unique equilibrium in the subgames with either order $1, 1, ...$ or order $2, 2, ...$ From now on, we are going to say "the equilibrium (payoff) associated with order $1, 1, ...$" instead of "the equilibrium (payoff) in the subgame with order $1, 1, ...$". By the definition of supremum $\bar{\pi}_B$, there exists a sequence of buyer's equilibrium payoffs $\{\pi_B^k\}_{k=1}^{\infty}$ that converges to $\bar{\pi}_B$. Pick any equilibrium payoff π_B^k from this sequence, and denote an associated equilibrium as E^k (there could be other equilibria yielding the same payoff π_B^k for the buyer) and the associated bargaining order as $i_1, i_2, i_3, ...$.

Given equilibrium E^k, denote $\hat{\pi}_B^k$ as the buyer's payoff in the subgame $\hat{\Gamma}(B, (i_2, i_3, ...))$. Since $\{\pi_B^k\}_{k=1}^{\infty}$ converges to the supremum payoff $\bar{\pi}_B$, it must have a subsequence $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ such that $\pi_B^{k_m} \geq \hat{\pi}_B^k$ for all m. There could be two cases: there is a payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ that is associated with an order starting with i_1, or there is no payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ that is associated with an order starting with i_1.

In the first case, we are going to construct an equilibrium, denoted as E^{*k} below, in an subgame with an order whose first *two* sellers are i_1 such that the buyer's payoff in E^{*k} is no less than that in E^k. Since the construction is relatively involved, it is helpful to explain its outline first. We first consider the buyer's payoff $\hat{\pi}_B^k$ in the subgame with order $i_2, i_3, ...$ in equilibrium E^k, then we find an equilibrium E^{k_j} in an subgame with order $i_1, i_2', i_3'...$ such that the buyer's payoff is no less than $\hat{\pi}_B^k$. Using the strategies in equilibrium E^{k_j}, we construct the equilibrium E^{*k} in the subgame with order $i_1, i_1, i_2', i_3'....$ The construction in the second case is similar.

In the first case, suppose there is a payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$, $\pi_B^{k_j}$, which is associated with an order starting with i_1. Given $\pi_B^{k_j}$, let E^{k_j} be an equilibrium associated with payoff $\pi_B^{k_j}$, and denote the order associated with this equilibrium as $i_1, i_2', i_3', ...$

Because of Claim 17, we can assume that the first agreement is reached in the first period in both E^k and E^{k_j} without loss of generality. Let us first examine the strategies in equilibrium E^k associated with order $i_1, i_2, i_3, ...$ In the first period of subgame $\hat{\Gamma}(i_1, (i_1, i_2, i_3, ...))$, seller i_1 suggests a price $q_{i_1}^k$ such that

$$\delta \left(1 - p^1(v_{\bar{i}_1})\right) - q_{i_1}^k = \delta \hat{\pi}_B^k, \tag{76}$$

where $p^1(v_{\bar{i}_1})$ is the price for the player other than i_1 and $\hat{\pi}_B^k$ is the buyer's payoff in the subgame $\hat{\Gamma}(B, (i_2, i_3, ...))$ according to equilibrium E^k. The price $q_{i_1}^k$ is accepted by the buyer. In the first period of subgame $\hat{\Gamma}(B, (i_1, i_2, i_3, ...))$, the buyer suggests a price $p_{i_1}^k$ such that seller i_1 is indifferent between accepting and rejecting,

$$p_{i_1}^k = H_{i_1,1} + \delta q_{i_1}^k, \tag{77}$$

and $p_{i_1}^k$ is also accepted by the seller.

An equilibrium, E^{*k}, with the order $i_1, i_1, i_2', i_3', ...$ is described as follows. The strategies in the subgame $\hat{\Gamma}(B, (i_1, i_2', i_3', ...))$ are the same as those in equilibrium E^{k_j}. In the subgame $\hat{\Gamma}(i_1, (i_1, i_1, i_2', i_3', ...))$, seller i_1 suggests a price $q_{i_1}^{*k}$ and it is accepted. In the subgame $\hat{\Gamma}(B, (i_1, i_1, i_2', i_3', ...))$, the buyer suggests a price $p_{i_1}^{*k}$ and it is also accepted. In particular, in the first period of $\hat{\Gamma}(i_1, (i_1, i_1, i_2', i_3', ...))$, seller i_1 suggests a price $q_{i_1}^{*k}$ such that

$$\delta \left(1 - p^1(v_{\tilde{\imath}_1})\right) - q_{i_1}^{*k} = \delta \pi_B^{k_j}. \tag{78}$$

In the first period of $\hat{\Gamma}(B, (i_1, i_1, i_2', i_3', ...))$, the buyer makes an offer such that seller i_1 is indifferent between accepting and rejecting and the seller accepts it, so

$$p_{i_1}^{*k} = H_{i_1,1} + \delta q_{i_1}^{*k}. \tag{79}$$

Now let us compare equilibria E^{k_j} and E^{*k}. By the definition of $\pi_B^{k_j}$, we have $\pi_B^{k_j} \geq \hat{\pi}_B^k$, therefore (76) and (78) imply that $q_{i_1}^{*k} \leq q_{i_1}^k$, which, combined with (77) and (79), gives $p_{i_1}^{*k} \leq p_{i_1}^k$. Notice that $p_{i_1}^{*k}$ and $p_{i_1}^k$ are accepted in E^k and E^{*k} respectively, so the buyer's payoff in E^{*k} is no less than that in E^k.

In the second case, each payoff in $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ has an equilibrium order that begins with $\tilde{\imath}_1 \neq i_1$. Since $\{\pi_B^{k_m}\}_{m=1}^{\infty}$ converges to the supremum by definition, there also exists a payoff $\pi_B'^{k_j}$ in this sequence that is no less than $\hat{\pi}_B^k$. Denote the order associated with $\pi_B'^{k_j}$ as $\tilde{\imath}_1, i_2'', i_3''...$ Analogous to the first case, there is an equilibrium with order $\tilde{\imath}_1, \tilde{\imath}_1, i_2'', i_3''...$ giving the buyer at least π_B^k.

In summary, we have shown that for any sequence of equilibrium payoffs converging to $\bar{\pi}_B$, there is another sequence that also converges to $\bar{\pi}_B$, and each equilibrium payoff in this sequence has the same first two elements in its bargaining order.

By induction, if there exists a sequence of equilibria with the first t elements being identical, whose buyer's payoffs converge to $\bar{\pi}_B$, there is another sequence of equilibria with identical first $t+1$ elements, where the buyer's payoffs converge to $\bar{\pi}_B$. Hence, $\bar{\pi}_B$ can be approached by equilibrium payoffs in the subgames with either order $1, 1, ...$ or order $2, 2, ...$ ∎

Claim 19 *Subgame $\hat{\Gamma}(B, (2, 2, ...))$ has a unique equilibrium with an agreement in the first period, while $\hat{\Gamma}(B, (1, 1, ...))$ either has no agreement or has a unique equilibrium with an agreement in the first period.*

Proof. The proof is similar to the analysis on the Rubinstein bargaining game.[30] In particular, the supremum of p_2 and the supremum of q_2 in the equilibria of $\hat{\Gamma}(B, (2, 2, ...))$ satisfy

[30] See Fudenberg and Tirole (1991), pp.115-116.

(18) and (19), so do the infimum of p_2 and the infimum of q_2. Therefore the supremum and infimum of p_2 coincide, which implies that the selling price for seller 2 is unique. As a result, the unique outcome is $\left(p_1^1, p_2^2, 2, 1\right)$, where p_2^2 is given in (13).

Given the bargaining order, seller 1 sells first, otherwise there is no agreement in $\hat{\Gamma}\left(B, (1,1,...)\right)$. Suppose seller 1 sells first. His price is at least v_1, so the buyer's payoff is at most $-v_1 - \delta p_2^1 + \delta$, which is negative when $\delta v_2 + (1+\delta) v_1 > \delta$. As a result, $\hat{\Gamma}\left(B, (1,1,...)\right)$ has no agreement if $\delta v_2 + (1+\delta) v_1 > \delta$. On the other hand, if $\delta v_2 + (1+\delta) v_1 \leq \delta$, analogous to $\hat{\Gamma}\left(B, (2,2,...)\right)$, subgame $\hat{\Gamma}\left(B, (1,1,...)\right)$ also has a unique equilibrium with an agreement in the first period. ■

Claim 20 *The buyer's equilibrium payoff in* $\hat{\Gamma}\left(B, (1,1,...)\right)$ *is less than* $\bar{\pi}_B$.

Proof. Subgame $\hat{\Gamma}\left(B, (2,2,...)\right)$ has a unique equilibrium according to Claim 19, and the equilibrium is given in Lemma 6. Therefore, the buyer's payoff is given by (20), which is positive when condition (16) holds.

Claim 19 implies that $\hat{\Gamma}\left(B, (1,1,...)\right)$ has either no agreement or a unique equilibrium with an agreement in the first period. If $\hat{\Gamma}\left(B, (1,1,...)\right)$ has no agreement, the buyer's payoff is zero. If $\hat{\Gamma}\left(B, (1,1,...)\right)$ has a unique equilibrium with an agreement in the first period, an analogue of (18) and (19) gives the unique selling price for seller 1, so the buyer's payoff is

$$\pi'_B = \frac{1}{1+\delta}\left[\delta \frac{1}{1+\delta}(1-v_2) - v_1\right],$$

which is also less than π_B^*. Hence, the buyer's equilibrium payoff in $\hat{\Gamma}\left(B, (1,1,...)\right)$ is less than π_B^*, so it is also less than $\bar{\pi}_B$. ■

Claim 21 *The buyer's equilibrium payoff is* $\bar{\pi}_B$ *only when the bargaining order is* $2, 2, ...$

Proof. Claims 18 to 20 imply that the buyer's equilibrium payoff in $\hat{\Gamma}\left(B, (2,2,...)\right)$ is $\bar{\pi}_B$, so it is sufficient to show that the buyer's equilibrium payoff given any other order is less than $\bar{\pi}_B$. Suppose there is an equilibrium, E_2 with another order $i_1, i_2, i_3, ...$ where the buyer's payoff is also $\bar{\pi}_B$. We are going to show that this assumption leads to a contradiction.

First consider the case with $i_1 = 2$. Since the order $i_1, i_2, ...$ is different to $2, 2, ...$, assume $i_2 = 1$ without loss of generality.[31] By the same reasoning in Claim 17, E_2 has an agreement in period 1. Since seller 2's price is no less than v_2, we have

$$\delta\left(1 - p_1^1\right) - v_2 \geq \bar{\pi}_B. \tag{80}$$

In the first period of $\hat{\Gamma}\left(2, (2,1,i_3,...)\right)$, seller 2 offers q_2'' such that

$$\delta\left(1 - p_1^1\right) - q_2'' = \delta \pi_B'', \tag{81}$$

[31] To see why it is without loss of generality to assume that i_2 is the first "1" in the order, consider, for example, the case where i_3 is the first "1" in the bargaining order. If the buyer's payoff is less than $\bar{\pi}_B$ in the subgame with order $i_2, i_3, ...$, then by backward induction her payoff is also less than $\bar{\pi}_B$ in the game with order $i_1, i_2, ...$

where π''_B is the payoff of $\hat{\Gamma}(B,(1,i_3,...))$ in E_2, and $q''_2 \geq v_2$ is guaranteed by

$$\delta\left(1-p_1^1\right)-v_2 \geq \bar{\pi}_B \geq \pi''_B,$$

where the first inequality is given by (80) and the second inequality comes from the definition of $\bar{\pi}_B$. Similarly, the buyer offers p''_2 such that seller 2 is indifferent between accepting and rejecting in the first period of $\hat{\Gamma}(B,(2,i_2,i_3,...))$, so

$$p''_2 = H_{2,1} + \delta q''_2. \tag{82}$$

In the unique equilibrium given the order $2,2,...$, the buyer offers p_2^2 and seller 2 offers q_2^2 according to (18) and (19) that can be rewritten as

$$\delta\left(1-p_1^1\right)-q_2^2 = \delta\bar{\pi}_B. \tag{83}$$

Claim 20 implies that $\pi''_B < \bar{\pi}_B$, so we have $q_2^2 < q''_2$ by comparing (81) and (83). Similarly, by comparing (18) and (82), we have $p_2^2 < p''_2$, hence the buyer's payoff in E_2 is less than $\bar{\pi}_B$. This is a contradiction to the definition of E_2.

Analogously, there is also a contradiction when $i_1 = 1$. Hence, if the bargaining order is different to $2,2,...$, the buyer's equilibrium payoff is less than $\bar{\pi}_B$. ■

Proof of Proposition 5. Claims 17 to 21 prove Proposition 5 in the two-seller game. Therefore, it remains to generalize the proof to the N-seller game. The arguments for Lemma 6 and Claims 17 to 21 analogously apply to the N-seller game with commitment. As a result, only a sketch of the proof is provided below.

The equilibrium price for seller N can be solved from (71), (72) and (73):

$$p_N^N = \frac{\delta}{1+\delta}\left(\left(\frac{\delta}{\delta+1}\right)^{N-1} - \sum_{i=1}^{N}\left(\frac{\delta}{1+\delta}\right)^{N-i} v_i\right). \tag{84}$$

As a result, condition (8) ensures that the mall is profitable for the buyer and seller N. Moreover, it is easy to verify that (8) also implies that any other seller's equilibrium price is also higher than the value of his land. Equation (84) gives the equilibrium price for seller N in the N-seller game, and it also gives any seller n's equilibrium price p_n^n in the N-seller game if N is replaced with n in the equation. Therefore, the unique equilibrium outcome is $\left(p_1^1, p_2^2, ..., p_N^N, N, N-1, ..., 1\right)$.

Following the same reasoning as in Claims 17 to 21 in the two-seller game, we can show that there is a unique equilibrium bargaining order $N, N, ...$ Moreover, it is only if the buyer chooses this order that her equilibrium payoff reaches the highest equilibrium payoff without commitment. ■